Particle motion in the background field

$$m_0 V_\vartheta = P_\vartheta - \frac{e}{c} A_{0\vartheta}(r)$$

$$m_0 V_z = P_z - \frac{e}{c} A_{0z}(r)$$

$$m_0 u_\parallel = m_0 \left(h^\vartheta V_\vartheta + h^z V_z \right) = h^\vartheta \left(P_\vartheta - \frac{e}{c} A_{0\vartheta} \right) + h^z \left(P_z - \frac{e}{c} A_{0z} \right)$$

$$H_0 = \frac{P_r^2}{2m_0} + \frac{1}{2m_0 r^2} \left(P_\vartheta - \frac{e}{c} A_{0\vartheta}(r) \right)^2 + \frac{1}{2m_0} \left(P_z - \frac{e}{c} A_{0z}(r) \right)^2 + e\Phi_0(r) =$$

$$= \frac{P_r^2}{2m_0} + U(r, P_\vartheta, P_z).$$

Gyrocenter: $\frac{\partial}{\partial r} U(r, P_\vartheta, P_z) = 0 \Rightarrow r = r_0(P_\vartheta, P_z).$

$$H_0 = \frac{P_r^2}{2m_0} + U_0 + \frac{1}{2!} U_0''(r - r_0)^2 + \frac{1}{3!} U_0'''(r - r_0)^3 + \dots$$

After transformation to $(\vec{J}, \vec{\theta})$ variables by canonical perturbation theory we get in the first order of Larmor radius expansion:

$$\begin{cases} H_0 = U_0 + \Omega_0 J_\perp, \quad U_0 = U(r_0), \quad \Omega_0 = \left(U_0''/m_0 \right)^{1/2}, \quad \rho = \left(\frac{2 J_\perp}{m_0 \Omega_0} \right)^{1/2} \\ r = r_0 - \rho \cos\phi \\ \vartheta = \vartheta_g + m_0 \Omega_0 \frac{\partial r_0}{\partial P_\vartheta} \rho \sin\phi \\ z = z_g + m_0 \Omega_0 \frac{\partial r_0}{\partial P_z} \rho \sin\phi \end{cases}$$

$$\Omega^\alpha = \frac{\partial H_0}{\partial J_\alpha}, \quad \vec{J} = (J_\perp, P_\vartheta, P_z), \quad \vec{\theta} = (\phi, \vartheta_g, z_g)$$

Although we can integrate over (P_ϑ, P_z) and find Ω^α, H_0, $r = r_0(P_\vartheta, P_z)$ numerically we currently transform to new variables: $(P_\vartheta, P_z) \to (r_0, u_\parallel)$ as follows and do all analytically retaining terms within the order of approximation.

$$U'(r_0, P_\vartheta, P_z) = -\frac{1}{m_0 r_0^3}\left(P_\vartheta - \frac{e}{c}A_{0\vartheta}(r_0)\right)^2 - \frac{\omega_c \hat{h}_z}{r_0}\left(P_\vartheta - \frac{e}{c}A_{0\vartheta}(r_0)\right) +$$
$$+ \omega_c \hat{h}_\vartheta\left(P_z - \frac{e}{c}A_{0z}\right) + e\Phi_0'(r_0) = 0. \qquad (*)$$

We used: $\vec{B}_0 = \nabla \times \vec{A}_0$, $A_{0\vartheta}' = \frac{c}{e} m_0 r_0 \omega_c \hat{h}_z$, $A_{0z}' = -\frac{c}{e} m_0 \omega_c \hat{h}_\vartheta$.

$$P_z - \frac{e}{c}A_{0z} = \frac{1}{h^z}\Big[m_0 u_\parallel - h^\vartheta \underbrace{\left(P_\vartheta - \frac{e}{c}A_{0\vartheta}\right)}_{X}\Big] = \frac{1}{h^z}\left[m_0 u_\parallel - h^\vartheta X\right].$$

(*):

$$X^2 + bX + c = 0; \quad b = \frac{m_0 r_0^2 \omega_c}{h_z}, \quad c = -\frac{m_0^2 r_0^3 \omega_c}{h_z}\left(\hat{h}_\vartheta u_\parallel + \hat{h}_z V_E\right).$$

$$X_{1,2} = \frac{1}{2}\left[-b \pm \sqrt{b^2 - 4c}\right] = \frac{b}{2}\left[-1 \pm \sqrt{1 - \frac{4c}{b^2}}\right], \quad h_z > 0.$$

$$-\frac{4c}{b^2} = \frac{4h_z}{r_0 \omega_c}\left[\hat{h}_\vartheta u_\parallel + \hat{h}_z V_E\right] \sim 4\hat{h}_\vartheta \,\rho/r_0 \ll 1. \qquad \boxed{V_E \ll u_\parallel \sim V_T}$$

$\hat{h}_\vartheta = 0$!!!

$$\sqrt{1+x} \approx 1 + \frac{x}{2} - \frac{x^2}{8}. \quad [\;] \approx \frac{x}{2}\left(1 - \frac{x}{4}\right), \quad x = -\frac{4c}{b^2}.$$

$$\begin{cases} P_\vartheta = \frac{e}{c}A_{0\vartheta} + m_0 h_\vartheta u_\parallel + m_0 V_{E\vartheta} - \frac{m_0 h_z}{\omega_c}\left(\hat{h}_\vartheta u_\parallel + \hat{h}_z V_E\right)^2 \\ P_z = \frac{e}{c}A_{0z} + m_0 h_z u_\parallel + m_0 V_{Ez} + \frac{m_0 h^\vartheta}{\omega_c}\left(\hat{h}_\vartheta u_\parallel + \hat{h}_z V_E\right)^2 \end{cases}$$

$$V_{E\vartheta} = r_0 h_z V_E, \quad V_E^\vartheta = \frac{h_z}{r_0} V_E; \quad V_{Ez} = V_E^z = -\frac{h_\vartheta}{r_0} V_E = -\hat{h}_\vartheta V_E.$$

$$V_0'' = \frac{3}{m_0 r_0^4}\left(P_\vartheta - \frac{e}{c}A_{0\vartheta}\right)^2 - \frac{2}{\cancel{m_0} r_0^{\cancel{3}}}\left(P_\vartheta - \frac{e}{c}A_{0\vartheta}\right)\cdot\left(-\frac{e}{\cancel{c}}\right)\cdot\frac{\cancel{c}}{e}\cancel{m_0}\cancel{r_0}\,\omega_c \hat{h}_z -$$

$$-\left(\frac{\omega_c \hat{h}_z}{r_0}\right)'\left(P_\vartheta - \frac{e}{c}A_{0\vartheta}\right) - \frac{\omega_c \hat{h}_z}{\cancel{r_0}}\left(-\frac{e}{\cancel{c}}\right)\cdot\frac{\cancel{c}}{e} m_0 \cancel{r_0}\,\omega_c \hat{h}_z +$$

$$+\left(\omega_c \hat{h}_\vartheta\right)'\left(P_z - \frac{e}{c}A_{0z}\right) + \omega_c \hat{h}_\vartheta\left(-\frac{e}{c}\right)\left(-\frac{c}{e}\right)\cdot m_0 \omega_c \hat{h}_\vartheta + e\,\phi_0''(r_0) =$$

$$= \frac{3}{m_0 r_0^4}\left(P_\vartheta - \frac{e}{c}A_{0\vartheta}\right)^2 + \frac{2\omega_c \hat{h}_z}{r_0^2}\left(P_\vartheta - \frac{e}{c}A_{0\vartheta}\right) - \left(\frac{\omega_c \hat{h}_z}{r_0}\right)'\left(P_\vartheta - \frac{e}{c}A_{0\vartheta}\right) +$$

$$+ m_0 \omega_c^2 \hat{h}_z^2 + \left(\omega_c \hat{h}_\vartheta\right)'\left(P_z - \frac{e}{c}A_{0z}\right) + m_0 \omega_c^2 \hat{h}_\vartheta^2 + e\,\phi_0'' =$$

$$\approx \underset{\sim \varepsilon^2}{\frac{3}{m_0 r_0^4} m_0^2 h_\vartheta^2 u_\parallel^2} + \left\{\frac{2\omega_c \hat{h}_z}{r_0^2} - \left(\frac{\omega_c \hat{h}_z}{r_0}\right)'\right\}\underset{\sim \varepsilon}{\left(m_0 h_\vartheta u_\parallel + m_0 V_{E\vartheta}\right)} +$$

$$+ \underset{\sim 1}{m_0 \omega_c^2} + \underset{\sim \varepsilon}{\left(\omega_c \hat{h}_\vartheta\right)'\left(m_0 h_z u_\parallel + m_0 V_{Ez}\right)} + \underset{\sim \varepsilon}{e\,\phi_0''}.$$

Currently we neglect all terms of ρ/r_0 order here and use:

$$V_0'' \approx m_0 \omega_c^2 (r_0).$$

We find now frequences:

$$\begin{cases} \Omega^{\phi} = \dfrac{\partial H_0}{\partial J_\perp} = \Omega_0 = \omega_c(r_0) = \omega_c(P_\vartheta, P_z) \\[2ex] \Omega^{\vartheta} = \dfrac{\partial H_0}{\partial P_\vartheta} = \dfrac{\partial U_0}{\partial P_\vartheta} + \dfrac{\partial \Omega_0}{\partial P_\vartheta} J_\perp \\[2ex] \Omega^{z} = \dfrac{\partial H_0}{\partial P_z} = \dfrac{\partial U_0}{\partial P_z} + \dfrac{\partial \Omega_0}{\partial P_z} J_\perp \end{cases}$$

$$\begin{cases} \dfrac{\partial U_0}{\partial P_\vartheta} = \dfrac{\partial U_0(r_0, P_\vartheta, P_z)}{\partial P_\vartheta} = \cancel{\dfrac{\partial U_0}{\partial r_0}}^{\,0} \dfrac{\partial r_0}{\partial P_\vartheta} + \dfrac{\partial U_0}{\partial P_\vartheta} = \dfrac{1}{m_0 r_0^2}\left(P_\vartheta - \frac{e}{c} A_{0\vartheta}\right) \\[2ex] \dfrac{\partial U_0}{\partial P_z} = \dfrac{\partial U_0(r_0, P_\vartheta, P_z)}{\partial P_z} = \cancel{\dfrac{\partial U_0}{\partial r_0}}^{\,0} \dfrac{\partial r_0}{\partial P_z} + \dfrac{\partial U_0}{\partial P_z} = \dfrac{1}{m_0}\left(P_z - \frac{e}{c} A_{0z}\right) \end{cases}$$

$$\begin{cases} \dfrac{\partial U_0}{\partial P_\vartheta} = \dfrac{1}{m_0 r_0^2}\left\{ m_0 h_\vartheta u_\parallel + m_0 V_{E\vartheta} - \dfrac{m_0 h_z}{\omega_c}\left(\hat{h}_\vartheta u_\parallel + \hat{h}_z V_E\right)^2 \right\} \\[2ex] \dfrac{\partial U_0}{\partial P_z} = \dfrac{1}{m_0}\left\{ m_0 h_z u_\parallel + m_0 V_{Ez} + \dfrac{m_0 h^\vartheta}{\omega_c}\left(\hat{h}_\vartheta u_\parallel + \hat{h}_z V_E\right)^2 \right\} \end{cases}$$

$$\frac{\partial \Omega_0}{\partial P_\vartheta} = \omega_c' \frac{\partial r_0}{\partial P_\vartheta}, \quad \frac{\partial \Omega_0}{\partial P_z} = \omega_c' \frac{\partial r_0}{\partial P_z}$$

To evaluate $\frac{\partial r_0}{\partial P_\alpha}$ consider $U'(r_0, P_\vartheta, P_z) = 0$:

$$\begin{cases} U_0'' \dfrac{\partial r_0}{\partial P_\vartheta} + \dfrac{\partial U_0'}{\partial P_\vartheta} = 0, & \dfrac{\partial U_0'}{\partial P_\vartheta} = -\dfrac{2}{m_0 r_0^3}\left(P_\vartheta - \frac{e}{c} A_{0\vartheta}\right) - \dfrac{\omega_c \hat{h}_z}{r_0} \approx -\dfrac{\omega_c \hat{h}_z}{r_0}. \\[2ex] U_0'' \dfrac{\partial r_0}{\partial P_z} + \dfrac{\partial U_0'}{\partial P_z} = 0, & \dfrac{\partial U_0'}{\partial P_z} = \omega_c \hat{h}_\vartheta. \end{cases}$$

$$\frac{\partial r_0}{\partial P_\vartheta} = +\frac{\omega_c \hat{h}_z}{r_0 m_0 \omega_c^{\cancel{2}}} = \frac{\hat{h}_z}{r_0 m_0 \omega_c}; \quad \frac{\partial r_0}{\partial P_z} = -\frac{\omega_c \hat{h}_\vartheta}{m_0 \omega_c^2} = -\frac{\hat{h}_\vartheta}{m_0 \omega_c}$$

$$\boxed{\frac{\partial r_0}{\partial P_\vartheta} = \frac{\hat{h}_z}{m_0 \omega_c r_0}, \quad \frac{\partial r_0}{\partial P_z} = -\frac{\hat{h}_\vartheta}{m_0 \omega_c}}$$

(5')

$$\begin{cases} \Omega^{\vartheta} = h^{\vartheta} u_{\parallel} + V_E^{\vartheta} - \dfrac{h_z}{\omega_c r_0^2} \left(\hat{h}_{\vartheta} u_{\parallel} + \hat{h}_z V_E \right)^2 + \dfrac{\hat{h}_z \omega_c'}{m_0 \omega_c r_0} J_{\perp} \\ \Omega^{z} = h^{z} u_{\parallel} + V_E^{z} + \dfrac{\hat{h}_{\vartheta}}{\omega_c r_0} \left(\hat{h}_{\vartheta} u_{\parallel} + \hat{h}_z V_E \right)^2 - \dfrac{\hat{h}_{\vartheta} \omega_c'}{m_0 \omega_c} J_{\perp} \end{cases}$$

$$\begin{cases} \Omega^{\vartheta} = h^{\vartheta} u_{\parallel} + V_E^{\vartheta} - \dfrac{h_z}{\omega_c} \left(h^{\vartheta} u_{\parallel} + V_E^{\vartheta} \right)^2 + \dfrac{\hat{h}_z \omega_c'}{m_0 \omega_c r_0} J_{\perp} \\ \Omega^{z} = h^{z} u_{\parallel} + V_E^{z} + \dfrac{h_{\vartheta}}{\omega_c} \left(h^{\vartheta} u_{\parallel} + V_E^{\vartheta} \right)^2 - \dfrac{\hat{h}_{\vartheta} \omega_c'}{m_0 \omega_c} J_{\perp} \end{cases}$$

Now consider Eq. (15) of KiLCA paper when $\frac{\partial}{\partial t} = -i\omega$ for $\tilde{Q}_{\vec{m}} = 0$

$$i \left(\vec{m} \cdot \vec{\Omega} - \omega \right) \tilde{f}_{\vec{m}} = \hat{L}_c \tilde{f}_m .$$

$$\vec{m} \cdot \vec{\Omega} - \omega = l\omega_c + k_{\vartheta} \Omega^{\vartheta} + k_z \Omega^z - \omega = l\omega_c + k_{\parallel} u_{\parallel} + k_{\perp} V_E -$$

$$- \frac{r_0 k_{\perp}}{\omega_c} \left(h^{\vartheta} u_{\parallel} + V_E^{\vartheta} \right)^2 + \frac{k_{\perp}}{m_0} \frac{\omega_c'}{\omega_c} J_{\perp} - \omega$$

$$\vec{m} \cdot \vec{\Omega} - \omega = l\omega_c + k_{\parallel} u_{\parallel} + k_{\perp} V_E - \frac{k_{\perp} r_0}{\omega_c} \left(h^{\vartheta} u_{\parallel} + V_E^{\vartheta} \right)^2 + \frac{k_{\perp}}{m_0} \frac{\omega_c'}{\omega_c} J_{\perp} -$$
$$- \omega .$$

Collision operator for drifting Maxwellian distribution:

$$\hat{L}_c \tilde{f}(\vec{J}) = \nu V_T^2 \frac{\partial}{\partial u_{\parallel}} \left(\frac{\partial}{\partial u_{\parallel}} + \frac{u_{\parallel} - V_{\parallel}}{V_T^2} \right) \tilde{f}(\vec{J})$$

$$\hat{L}_c f_0(\vec{J}) = 0 .$$

Kinetic equation: We introduce a new velocity

$$u = \frac{u_\parallel - V_\parallel}{V_T}, \quad u_\parallel = V_T u + V_\parallel, \quad \frac{\partial}{\partial u_\parallel} = \frac{1}{V_T}\frac{\partial}{\partial u}.$$

$$\hat{L}_c \tilde{f}(\vec{y}) = \nu V_T^2 \frac{1}{V_T}\frac{\partial}{\partial u}\left(\frac{1}{V_T}\frac{\partial}{\partial u} + \frac{u}{V_T}\right)\tilde{f} = \nu \frac{\partial}{\partial u}\left[\frac{\partial}{\partial u} + u\right]\tilde{f}(\vec{y}).$$

$$\vec{m}\cdot\vec{\Omega} - \omega = \ell\omega_c + k_\parallel (V_T u + V_\parallel) + k_\perp V_E - \frac{k_\perp r_0}{\omega_c}\left\{(h^\vartheta)^2 (V_T u + V_\parallel)^2 + \right.$$

$$\left. + 2h^\vartheta (V_T u + V_\parallel) V_E^\vartheta + (V_E^\vartheta)^2\right\} + \frac{k_\perp}{m_0}\frac{\omega_c'}{\omega_c} y_\perp - \omega =$$

$$= \ell\omega_c + k_\parallel V_\parallel + k_\perp V_E - \omega + \frac{k_\perp}{m_0}\frac{\omega_c'}{\omega_c} y_\perp - \frac{k_\perp r_0}{\omega_c}\left\{(h^\vartheta)^2 V_\parallel^2 + \right.$$

$$\left. + 2h^\vartheta V_\parallel V_E^\vartheta + (V_E^\vartheta)^2\right\} + \left\{k_\parallel V_T - \frac{k_\perp r_0}{\omega_c}\left[(h^\vartheta)^2 2V_T V_\parallel + 2h^\vartheta V_T V_E^\vartheta\right]\right\}u$$

$$- \frac{k_\perp r_0}{\omega_c}\left\{(h^\vartheta)^2 V_T^2\right\}u^2 = -\hat{X}_0 + \hat{X}_1 u - \hat{X}_2 u^2$$

$$\frac{\partial}{\partial u}\left[\frac{\partial}{\partial u} + u\right]\tilde{f}_{\vec{m}} - \frac{i}{\nu}\left(-\hat{X}_0 + \hat{X}_1 u - \hat{X}_2 u^2\right) f_{\vec{m}} = 0$$

$$\frac{\partial}{\partial u}\left[\frac{\partial}{\partial u} + u\right]\tilde{f}_{\vec{m}} + i\left(\underbrace{\frac{\hat{X}_0}{\nu}}_{X_2} - \underbrace{\frac{\hat{X}_1}{\nu}}_{X_1}u + \underbrace{\frac{\hat{X}_2}{\nu}}_{X_3}u^2\right)\tilde{f}_{\vec{m}} = 0$$

$$\frac{\partial}{\partial u}\left[\frac{\partial}{\partial u} + u\right]\tilde{f}_{\vec{m}} + i\left(X_2 - X_1 u + X_3 u^2\right)\tilde{f}_{\vec{m}} = 0$$

$$\begin{cases} X_2 = \dfrac{1}{\nu}\left\{\omega - \ell\omega_c - k_\parallel V_\parallel - k_\perp V_E - \dfrac{k_\perp}{m_0}\dfrac{\omega_c'}{\omega_c} y_\perp + \dfrac{k_\perp r_0}{\omega_c}\left\{(h^\vartheta V_\parallel + V_E^\vartheta)^2\right\}\right\} \\[2ex] X_1 = \dfrac{1}{\nu}\left[k_\parallel V_T - \dfrac{k_\perp r_0}{\omega_c} 2h^\vartheta V_T (h^\vartheta V_\parallel + V_E^\vartheta)\right] \\[2ex] X_3 = \dfrac{1}{\nu}\dfrac{k_\perp r_0}{\omega_c}(h^\vartheta)^2 V_T^2. \end{cases}$$

In final form:

$$\frac{\partial}{\partial u}\left[\frac{\partial}{\partial u}+u\right]\tilde{f}_{\vec{m}}+i\left(X_2-X_1u+X_3u^2\right)\tilde{f}_m=0,$$

$$\begin{cases} X_2=\dfrac{\omega-\ell\omega_c-k_{\parallel}V_{\parallel}-k_{\perp}V_E}{\nu}-\dfrac{k_{\perp}\omega_c'}{\nu m_0\omega_c}\mathcal{Y}_{\perp}+\dfrac{k_{\perp}v_0}{\nu\omega_c}\left(h^{\vartheta}V_{\parallel}+V_E^{\vartheta}\right)^2 \\ X_1=\dfrac{k_{\parallel}V_T}{\nu}-2\dfrac{k_{\perp}V_T\hat{h}_{\vartheta}}{\nu\omega_c}\left(h^{\vartheta}V_{\parallel}+V_E^{\vartheta}\right) \\ X_3=\dfrac{k_{\perp}\hat{h}_{\vartheta}^2V_T^2}{\nu r_0\omega_c} \end{cases}$$

We can easily check that due to additionall terms all coefficeents (X_1, X_2, X_3) are invariant with respect to Galilean transformations.

The Green function

In KiLCA paper we got the following solution: $[\hat u = u_\parallel - V_\parallel]$

$$\left[\frac{\partial}{\partial t} + i(\omega_e + k_\parallel \hat u) - D\frac{\partial}{\partial \hat u}\left(\frac{\partial}{\partial \hat u} + \frac{\hat u}{V_T^2}\right)\right]\tilde f_{\vec m}(\hat u, t) = \tilde Q_{\vec m}(\hat u, t)$$

$$\tilde f_{\vec m}(\hat u, t) = \int_0^{t-t_0} d\tau \int_{-\infty}^{+\infty} d\hat u' \, \hat G(\hat u, \hat u', \tau)\tilde Q_{\vec m}(\hat u', t-\tau)$$

$$\hat G(\hat u, \hat u', \tau) = \frac{1}{\sqrt{4\pi\tilde a}}\exp\left\{ i\frac{k_\parallel}{\nu}(\hat u - \hat u') - c - \frac{1}{4\tilde a}\left(\hat u - \hat u' e^{-\nu\tau} + i\tilde b\right)^2\right\}$$

$$\tilde a(\tau) = \frac{V_T^2}{2}\left(1 - e^{-2\nu\tau}\right), \quad \tilde b(\tau) = \frac{2k_\parallel V_T^2}{\nu}\left(1 - e^{-\nu\tau}\right), \quad c(\tau) = \left(i\omega_e + \frac{k_\parallel^2 V_T^2}{\nu}\right)\tau$$

Let we introduce $u = \frac{\hat u}{V_T}$, $\hat u = u V_T$.

$$\left[\frac{\partial}{\partial t} + i(\omega_e + k_\parallel V_T u) - \nu \cancel{V_T^2}\frac{\partial^2}{\partial u^2}\frac{1}{\cancel{V_T^2}} - \nu\cancel{V_T^2}\frac{1}{\cancel{V_T}}\frac{\partial}{\partial u}\frac{u\cancel{V_T}}{\cancel{V_T^2}}\right]\tilde f_{\vec m}(u,t) = \tilde Q_{\vec m}(u,t)$$

$$\left[\frac{\partial}{\partial t} + i(\omega_e + k_\parallel V_T u) - \nu\frac{\partial}{\partial u}\left(\frac{\partial}{\partial u} + u\right)\right]\tilde f_{\vec m}(u,t) = \tilde Q_{\vec m}(u,t)$$

$$\left[-\frac{\partial}{\partial \nu t} - i\overset{-\tilde x_2}{\frac{\omega_e}{\nu}} - i\overset{x_1}{\frac{k_\parallel V_T}{\nu}}u + \frac{\partial}{\partial u}\left(\frac{\partial}{\partial u} + u\right)\right]\tilde f_{\vec m}(u,t) = -\frac{\tilde Q_m(u,t)}{\nu}$$

$$\tilde t = \nu t, \quad x_1 = \frac{k_\parallel V_T}{\nu}, \quad \tilde x_2 = -\frac{\omega_e}{\nu} = -\frac{l\omega_c + \omega_E + k_\parallel V_\parallel}{\nu}$$

$$\frac{\partial}{\partial u}\left(\frac{\partial}{\partial u} + u\right)\tilde f_{\vec m}(u,t) + \left[i\tilde x_2 - i x_1 u\right]\tilde f_{\vec m}(u,t) - \frac{1}{\nu}\frac{\partial \tilde f_{\vec m}}{\partial t} = -\frac{\tilde Q_m(u,t)}{\nu} = \tilde{\tilde Q}$$

This equation has form similar to eq. on page 7.
Let we write the solution in new variables:

$$\tilde f_{\vec m}(u,t) = \int_0^{t-t_0} d\tau \int_{-\infty}^{+\infty} du' v_T \, \hat G(u,u',\tau)\tilde Q_{\vec m}(u', t-\tau) = \left[\tilde Q = -\nu\tilde{\tilde Q}\right]$$

$$= -v_T\int_0^{t-t_0} d(\nu\tau)\int_{-\infty}^{+\infty} du' \, \hat G(u,u',\tau)\tilde{\tilde Q}(u', t-\tau)$$

$$\boxed{\tilde{\tau} = \nu\tau}$$

$$\hat{G}(u,u';\tilde{\tau}) = \frac{1}{\sqrt{\cancel{4}\pi \frac{v_T^2}{\cancel{2}}\left(1-e^{-2\tilde{\tau}}\right)}} \exp\left\{ i\,\frac{k_\parallel v_T}{\nu}(u-u') - \left(i\omega_e + \frac{k_\parallel^2 v_T^2}{\nu}\right)\frac{\tilde{\tau}}{\nu} - \right.$$

$$\left. - \frac{1}{\cancel{4}\frac{v_T^2}{\cancel{2}}\left(1-e^{-2\tilde{\tau}}\right)} \left[v_T u - v_T u' e^{-\tilde{\tau}} + i\,\frac{2k_\parallel v_T^2}{\nu}\left(1-e^{-\tilde{\tau}}\right)\right]^2 \right\} =$$

$$= \frac{1}{\sqrt{2\pi}\,v_T}\,\frac{1}{\left(1-e^{-2\tilde{\tau}}\right)^{1/2}} \exp\left\{ i x_1(u-u') + i\tilde{x}_2\tilde{\tau} - x_1^2\tilde{\tau} - \right.$$

$$\left. - \frac{1}{2\left(1-e^{-2\tilde{\tau}}\right)} \left[u - u' e^{-\tilde{\tau}} + 2i x_1\left(1-e^{-\tilde{\tau}}\right)\right]^2 \right\}$$

$$\tilde{f}_{\vec{m}}(u,t) = -\frac{\cancel{v_T}}{\sqrt{2\pi}\,\cancel{v_T}} \int_0^{\nu(t-t_0)} d\tilde{\tau} \int_{-\infty}^{+\infty} du'\, \tilde{\tilde{Q}}\left(u', \frac{\tilde{t}-\tilde{\tau}}{\nu}\right) \frac{1}{\left(1-e^{-2\tilde{\tau}}\right)^{1/2}} \exp\{\dots\}$$

We have $e^{-i\omega t}$ as time dependence in $\tilde{\tilde{Q}}$. Therefore $t_0 \to -\infty$ and

$$\tilde{f}_{\vec{m}}(u,t) = -\frac{1}{\sqrt{2\pi}} \int_0^{\infty} d\tilde{\tau} \int_{-\infty}^{+\infty} du'\, \tilde{\tilde{Q}}_a(u')\, e^{-i\omega\left(t - \frac{\tilde{\tau}}{\nu}\right)} \frac{1}{\left(1-e^{-2\tilde{\tau}}\right)^{1/2}} \exp\{\dots\}$$

In the following we take $e^{-i\omega t}$ out and consider Fourier amplitudes only.

$$f_m(u) = -\frac{1}{\sqrt{2\pi}} \int_0^{\infty} d\tilde{\tau} \int_{-\infty}^{+\infty} du'\, \tilde{\tilde{Q}}_a(u') \left(1-e^{-2\tilde{\tau}}\right)^{-1/2} \exp\left\{ i x_1(u-u') + i\underbrace{\left(\tilde{x}_2 + \frac{\omega}{\nu}\right)}_{x_2}\tilde{\tau} - \right.$$

$$\left. - x_1^2\tilde{\tau} - \frac{1}{2\left(1-e^{-2\tilde{\tau}}\right)}\left[u - u' e^{-\tilde{\tau}} + 2i x_1\left(1-e^{-\tilde{\tau}}\right)\right]^2 \right\}$$

Finally, solution of KE:

$$\frac{\partial}{\partial u}\left(\frac{\partial}{\partial u}+u\right) f_{\vec m}(u)+i(X_2-X_1 u) f_{\vec m}(u)=Q_{\vec m}(u)$$

$$f_{\vec m}(u)=\int_{-\infty}^{+\infty} du'\, G(u,u')\, Q_{\vec m}(u')$$

$$G(u,u')=-\frac{1}{\sqrt{2\pi}}\int_0^{\infty} d\tau\,\left(1-e^{-2\tau}\right)^{-1/2} \exp\Big\{ i X_1(u-u')+(iX_2-X_1^2)\tau -$$
$$-\frac{1}{2}\left(1-e^{-2\tau}\right)^{-1}\left[u-u'e^{-\tau}+2iX_1\left(1-e^{-\tau}\right)\right]^2\Big\}.$$

$$Q_{\vec m}(u)=-\frac{1}{\nu}\tilde{Q}_m(u,t)\,e^{i\omega t}, \quad u=\frac{u_\parallel-V_\parallel}{V_T}$$

$$X_1=\frac{k_\parallel V_T}{\nu}, \quad X_2=\frac{\omega-\omega_e}{\nu}=\frac{\omega-\ell\omega_c-k_\perp V_E-k_\parallel V_\parallel}{\nu}.$$

$$W_e^{mn}=\int_{-\infty}^{+\infty} du\,(u)^m f_{\vec m}(u), \quad \text{where} \quad Q_{\vec m}(u')=e^{-\frac{1}{2}\left(\frac{u'}{V_T}\right)^2}(u')^n$$

In our variables:

$$W_e^{mn}=\int_{-\infty}^{+\infty} du\, V_T\,(V_T u)^m \int_{-\infty}^{+\infty} du'\, G(u,u')\left(-\frac{1}{\nu}\right) e^{-\frac{1}{2}(u')^2}(u'V_T)^n=$$

$$=-\frac{(V_T)^{m+n+1}}{\nu}\int_{-\infty}^{+\infty} du\int_{-\infty}^{+\infty} du'\, u^m\, G(u,u')(u')^n e^{-\frac{1}{2}(u')^2}=$$

$$=-\frac{(V_T)^{m+n+1}}{\nu}\hat{I}^{mn}$$

$$\boxed{G_{IVAN}=-\left(\frac{1}{\sqrt{2\pi}}\right)^{-1} G_{SERGEY} \qquad \hat{I}^{mn}_{IVAN}=-\left(\frac{1}{\sqrt{2\pi}}\right)^{-1} I^{mn}_{SERGEY}}\ ?$$

$$\boxed{\hat{I}^{mn}=\int_{-\infty}^{+\infty} du\int_{-\infty}^{+\infty} du'\, u^m\, G(u,u')\, e^{-\frac{1}{2}(u')^2}(u')^n.}$$

$$\hat{I}^{00} = \int_{-\infty}^{+\infty} du \int_{-\infty}^{+\infty} du' \, G(u,u') \, e^{-\frac{1}{2}(u')^2} =$$

$$= -\sqrt{2\pi} \int_0^\infty d\tau \, e^{\tau(ix_2 - x_1^2) + x_1^2(1-e^{-\tau})} =$$

$$= -\sqrt{2\pi} \, e^{x_1^2} (x_1^2)^{ix_2 - x_1^2} \left(\Gamma(x_1^2 - ix_2) - \Gamma(x_1^2 - ix_2, x_1^2) \right) =$$

$$= -\sqrt{2\pi} \, {}_1F_1(1, 1 + x_1^2 - ix_2, x_1^2) / (x_1^2 - ix_2).$$

$$W_e^{00} = -\frac{v_T}{\nu} (-) \sqrt{2\pi} \, {}_1F_1 / (x_1^2 - ix_2)$$ — agrees with old expression!

Green function with magnetic drifts

With magnetic drifts KE reads: (see page 7)

$$\frac{\partial}{\partial u}\left[\frac{\partial}{\partial u}+u\right] f_{\vec m}(u) + i\left(x_2 - x_1 u + x_3 u^2\right) f_{\vec m}(u) = Q_{\vec m}(u)$$

$$Q_{\vec m}(u) = -\frac{1}{\nu}\tilde Q_{\vec m}(u,t)\, e^{i\omega t}, \quad u = \frac{u_{\parallel}-V_{\parallel}}{V_T}, \quad \tilde f_{\vec m}(u,t) = f_{\vec m}(u)\, e^{-i\omega t}.$$

We make substitution: $f_{\vec m}(u) = g(u)\, e^{\frac{\alpha u^2}{2}}$

$$\frac{\partial}{\partial u} f_{\vec m} = g' e^{\frac{\alpha u^2}{2}} + g e^{\frac{\alpha u^2}{2}}\alpha u = e^{\frac{\alpha u^2}{2}}\left(g' + \alpha u g\right)$$

$$\frac{\partial^2}{\partial u^2} f_{\vec m} = e^{\frac{\alpha u^2}{2}}\cdot \alpha u\left(g' + \alpha u g\right) + e^{\frac{\alpha u^2}{2}}\left(g'' + \alpha g + \alpha u g'\right) =$$

$$= e^{\frac{\alpha u^2}{2}}\left[g'' + 2\alpha u g' + g\left(\alpha^2 u^2 + \alpha\right)\right].$$

$$e^{\frac{\alpha u^2}{2}}\left\{g'' + 2\alpha u g' + g\left(\alpha^2 u^2 + \alpha\right) + g + u\left(g' + \alpha u g\right) + i\left(x_2 - x_1 u + x_3 u^2\right) g\right\} = Q_{\vec m}(u)$$

$$\cancel{g'' + g'(2\alpha u + u) + g\left[u^2(\alpha^2 + \alpha + i x_3) + u(-) i x_1 + \alpha + 1 + i x_2\right.}$$

$$g'' + g'(2\alpha u + u) + g\left(\alpha^2 u^2 + \alpha + 1 + \alpha u^2 + i x_2 - i x_1 u + i x_3 u^2\right) = Q_{\vec m}(u)\, e^{-\frac{\alpha u^2}{2}}$$

$$\alpha^2 + \alpha + i x_3 = 0 \Rightarrow \alpha = \sqrt{\frac{1}{4} - i x_3} - \frac{1}{2}.$$

$$g'' + g'(2\alpha + 1) u + g\left(\alpha + 1 + i x_2 - i x_1 u\right) = e^{-\frac{\alpha u^2}{2}} Q_{\vec m}(u)$$

New variable: $u = \frac{\bar u}{\sqrt{1 + 2\alpha}}$, $\bar u = u\sqrt{1 + 2\alpha}$

$$\frac{\partial}{\partial u} = \sqrt{1 + 2\alpha}\,\frac{\partial}{\partial \bar u}$$

$$(1+2\alpha)\frac{\partial^2 g}{\partial \bar{u}^2} + \cancel{\sqrt{1+2\alpha}}\,\frac{\partial g}{\partial \bar{u}}\cdot(1+2\alpha)\cdot\frac{\bar{u}}{\cancel{\sqrt{1+2\alpha}}} + g\Big(1+\alpha+ix_2 - ix_1\frac{\bar{u}}{\sqrt{1+2\alpha}}\Big) = e^{-\frac{\alpha}{2}\frac{\bar{u}^2}{(1+2\alpha)}}\,Q_{\vec{m}}(\bar{u})$$

$$\frac{\partial^2 g}{\partial \bar{u}^2} + \frac{\partial g}{\partial \bar{u}}\cdot\bar{u} + g\Big(1+2\alpha-\alpha+ix_2-ix_1\frac{\bar{u}}{\sqrt{1+2\alpha}}\Big)(1+2\alpha)^{-1} = \frac{e^{-\frac{\alpha}{2}\frac{\bar{u}^2}{(1+2\alpha)}}}{1+2\alpha}\,Q_{\vec{m}}(\bar{u})$$

$$\frac{\partial}{\partial \bar{u}}\Big(\frac{\partial}{\partial \bar{u}}+\bar{u}\Big)g + g\Big(\frac{ix_2-\alpha}{1+2\alpha} - \frac{ix_1\bar{u}}{(1+2\alpha)^{3/2}}\Big) = \frac{e^{-\frac{\alpha}{2}\frac{\bar{u}^2}{(1+2\alpha)}}}{(1+2\alpha)}\,Q_{\vec{m}}(\bar{u})$$

$$\frac{\partial}{\partial \bar{u}}\Big(\frac{\partial}{\partial \bar{u}}+\bar{u}\Big)g(\bar{u}) + g(\bar{u})\Big(i(\bar{x}_2-\bar{x}_1\bar{u})\Big) = \bar{Q}_{\vec{m}}(\bar{u})$$

$$\bar{x}_2 = \frac{x_2+i\alpha}{1+2\alpha}, \quad \bar{x}_1 = \frac{x_1}{(1+2\alpha)^{3/2}}, \quad \bar{Q}_{\vec{m}}(\bar{u}) = \frac{e^{-\frac{\alpha}{2}\frac{\bar{u}^2}{(1+2\alpha)}}}{(1+2\alpha)}\,Q_{\vec{m}}(\bar{u})$$

$$g(\bar{u}) = \int_{-\infty}^{+\infty} d\bar{u}'\, G(\bar{u},\bar{u}')\,\bar{Q}_{\vec{m}}(\bar{u}')$$

$$f_{\vec{m}}(\bar{u}) = g(\bar{u})\,e^{\frac{\alpha}{2}\frac{\bar{u}^2}{(1+2\alpha)}}$$

W^{mn} function

By definition: $W^{mn} = \int\limits_{-\infty}^{+\infty} du\, (u)^m f_{\vec m}(u)$,

where $f_{\vec m}(u)$ is solution of KE with $Q_{\vec m}(u) = u^n e^{-\frac{1}{2}\left(\frac{u}{V_T}\right)^2}$

and $\underset{\text{old !!!}}{\underline{u = u_{\parallel} - V_{\parallel}}}$. In case of magnetic drifts we have

natural variable $\bar u = \frac{u}{V_T}\sqrt{1+2\alpha}$, $u = \frac{V_T}{\sqrt{1+2\alpha}}\bar u$

$$f_{\vec m}(\bar u) = g(\bar u)\cdot e^{\frac{\alpha}{2}\frac{\bar u^2}{(1+2\alpha)}},$$

where $g(\bar u) = \int\limits_{-\infty}^{+\infty} d\bar u'\, G(\bar u, \bar u')\, \overline{Q}_{\vec m}(\bar u')$,

$$\overline{Q}_{\vec m}(\bar u) = \frac{e^{-\frac{\alpha}{2}\frac{\bar u^2}{(1+2\alpha)}}}{1+2\alpha}\left(-\frac{1}{\nu}\right)\cdot\left\{\left(\frac{V_T}{\sqrt{1+2\alpha}}\bar u\right)^n e^{-\frac{1}{2}\frac{\bar u^2}{1+2\alpha}}\right\}$$

$$W^{mn} = \int\limits_{-\infty}^{+\infty}\frac{V_T}{\sqrt{1+2\alpha}}d\bar u\left(\frac{V_T\bar u}{\sqrt{1+2\alpha}}\right)^m e^{\frac{\alpha}{2}\frac{\bar u^2}{(1+2\alpha)}}\int\limits_{-\infty}^{+\infty} d\bar u'\, G(\bar u,\bar u')\times$$

$$\times\left(-\frac{1}{\nu}\right)\left(\frac{V_T}{\sqrt{1+2\alpha}}\right)^n \bar u'^n\, \frac{e^{-\frac{\alpha}{2}\frac{\bar u'^2}{(1+2\alpha)} - \frac{1}{2}\frac{\bar u'^2}{1+2\alpha}}}{1+2\alpha} =$$

$$= (1+2\alpha)^{-\frac{1}{2}-\frac{m}{2}-\frac{n}{2}-1}\left(-\frac{1}{\nu}\right)V_T^{1+m+n}\int\limits_{-\infty\sqrt{1+2\alpha}}^{+\infty\sqrt{1+2\alpha}} d\bar u \int\limits_{-\infty}^{+\infty} d\bar u'\,(\bar u)^m\times$$

$$\times G(\bar u,\bar u')(\bar u')^n \exp\left\{\frac{\alpha}{2}\frac{\bar u^2-\bar u'^2}{(1+2\alpha)} - \frac{1}{2}\frac{\bar u'^2}{1+2\alpha}\right\}.$$

$$\{\ \} = \frac{\alpha\bar u^2 - (\alpha+1)\bar u'^2}{2(1+2\alpha)} = \frac{\alpha\bar u^2 - (1+2\alpha)\bar u'^2 + \alpha\bar u'^2}{2(1+2\alpha)} =$$

$$= -\frac{1}{2}\bar u'^2 + \frac{\alpha(\bar u^2 + \bar u'^2)}{2(1+2\alpha)}$$

$$W^{mn} = (1+2\alpha)^{-\frac{m+n+3}{2}} v_T^{m+n+1} \left(-\frac{1}{\nu}\right) \times$$

$$\int_{-\infty}^{+\infty} d\bar{u} \int_{-\infty}^{+\infty} d\bar{u}' \, (\bar{u})^m (\bar{u}')^n \, G(\bar{u},\bar{u}') \, e^{-\frac{1}{2}(\bar{u}')^2} \, e^{\frac{\alpha}{2}\frac{\bar{u}^2+\bar{u}'^2}{(1+2\alpha)}} .$$

$$W^{mn} = v_T^{m+n+1} \left(-\frac{1}{\nu}\right) \hat{I}_M^{mn}, \qquad \boxed{\gamma = \frac{\alpha}{1+2\alpha}}$$

$$\hat{I}_M^{mn} = (1+2\alpha)^{-\frac{m+n+3}{2}} \int_{-\infty}^{+\infty} du_1 \int_{-\infty}^{+\infty} du_2 \, u_1^m u_2^n \, G(u_1,u_2) \, e^{-\frac{u_2^2}{2} + \frac{\gamma}{2}(u_1^2+u_2^2)} =$$

$$= (1+2\alpha)^{-\frac{m+n+3}{2}} \frac{\partial^m}{\partial x^m} \frac{\partial^n}{\partial y^n} \int_{-\infty}^{+\infty} du_1 \int_{-\infty}^{+\infty} du_2 \, G(u_1,u_2) \, e^{x u_1 + y u_2 - \frac{u_2^2}{2} + \frac{\gamma}{2}(u_1^2+u_2^2)} .$$

$$G(u_1,u_2) = -\frac{1}{\sqrt{2\pi}} \int_0^{\infty} d\tau \left(1-e^{-2\tau}\right)^{-\frac{1}{2}} \exp\Big\{ i\bar{x}_1(u_1-u_2) + (i\bar{x}_2 - \bar{x}_1^2)\tau -$$

$$- \frac{1}{2}\left(1-e^{-2\tau}\right)^{-1} \left[u_1 - u_2 e^{-\tau} + 2i\bar{x}_1(1-e^{-\tau})\right]^2 \Big\}.$$

We have generalized Gaussian quadrature (2D) over (u_1,u_2) which can be evaluated analytically. Further computations are done in Mathematica. The answer is:

In[407]:=
```
KerR
I00 = II[0, 0]
I01 = II[0, 1]
I02 = II[0, 2]
I03 = II[0, 3]
I11 = II[1, 1]
I12 = II[1, 2]
I13 = II[1, 3]
I22 = II[2, 2]
I23 = II[2, 3]
I33 = II[3, 3]
```

Out[407]=
$$-\left(2\,e^{\frac{(-1+e^{\mathrm{tau}})(-1+2k)\,x1^2}{e^{\mathrm{tau}}(-1+k)+k}+\mathrm{tau}\,(-x1^2+i\,x2)+\frac{i\,(-1+e^{\mathrm{tau}})\,x1\,(x+y)}{e^{\mathrm{tau}}(-1+k)+k}+\frac{2\,e^{\mathrm{tau}}\,x\,y-e^{2\,\mathrm{tau}}(-1+k)\,(x^2+y^2)+k\,(x^2-y^2)}{2\,(e^{2\,\mathrm{tau}}(-1+k)^2-k^2)}}\sqrt{\pi}\right)\Big/\left(\sqrt{2-4\,k+(2-2\,e^{-2\,\mathrm{tau}})\,k^2}\right)$$

Out[408]=
$$-\left(2\,e^{\frac{(-1+e^{\mathrm{tau}})(-1+2k)\,x1^2}{e^{\mathrm{tau}}(-1+k)+k}-\mathrm{tau}\,(x1^2-i\,x2)}\sqrt{\pi}\right)\Big/\left(\sqrt{2-4\,k+(2-2\,e^{-2\,\mathrm{tau}})\,k^2}\right)$$

Out[409]=
$$-\frac{2\,i\,e^{\frac{(-1+e^{\mathrm{tau}})(-1+2k)\,x1^2}{e^{\mathrm{tau}}(-1+k)+k}-\mathrm{tau}\,(x1^2-i\,x2)}\,(-1+e^{\mathrm{tau}})\,\sqrt{\pi}\,x1}{(e^{\mathrm{tau}}\,(-1+k)+k)\,\sqrt{2-4\,k+(2-2\,e^{-2\,\mathrm{tau}})\,k^2}}$$

Out[410]=
$$\frac{e^{\frac{(-1+e^{\mathrm{tau}})(-1+2k)\,x1^2}{e^{\mathrm{tau}}(-1+k)+k}-\mathrm{tau}\,(x1^2-i\,x2)}\,(e^{2\,\mathrm{tau}}\,(-1+k)-k)\,\sqrt{2\,\pi}}{(e^{\mathrm{tau}}\,(-1+k)-k)\,(e^{\mathrm{tau}}\,(-1+k)+k)\,\sqrt{1-2\,k+k^2-e^{-2\,\mathrm{tau}}\,k^2}}+$$
$$\frac{e^{\frac{(-1+e^{\mathrm{tau}})(-1+2k)\,x1^2}{e^{\mathrm{tau}}(-1+k)+k}-\mathrm{tau}\,(x1^2-i\,x2)}\,(-1+e^{\mathrm{tau}})^2\,\sqrt{2\,\pi}\,x1^2}{(e^{\mathrm{tau}}\,(-1+k)+k)^2\,\sqrt{1-2\,k+k^2-e^{-2\,\mathrm{tau}}\,k^2}}$$

Out[411]=
$$\frac{3\,i\,e^{\frac{(-1+e^{\mathrm{tau}})(-1+2k)\,x1^2}{e^{\mathrm{tau}}(-1+k)+k}-\mathrm{tau}\,(x1^2-i\,x2)}\,(-1+e^{\mathrm{tau}})\,(e^{2\,\mathrm{tau}}\,(-1+k)-k)\,\sqrt{2\,\pi}\,x1}{(e^{\mathrm{tau}}\,(-1+k)-k)\,(e^{\mathrm{tau}}\,(-1+k)+k)^2\,\sqrt{1-2\,k+k^2-e^{-2\,\mathrm{tau}}\,k^2}}+$$
$$\frac{i\,e^{\frac{(-1+e^{\mathrm{tau}})(-1+2k)\,x1^2}{e^{\mathrm{tau}}(-1+k)+k}-\mathrm{tau}\,(x1^2-i\,x2)}\,(-1+e^{\mathrm{tau}})^3\,\sqrt{2\,\pi}\,x1^3}{(e^{\mathrm{tau}}\,(-1+k)+k)^3\,\sqrt{1-2\,k+k^2-e^{-2\,\mathrm{tau}}\,k^2}}$$

Out[412]=
$$-\frac{e^{\mathrm{tau}+\frac{(-1+e^{\mathrm{tau}})(-1+2k)\,x1^2}{e^{\mathrm{tau}}(-1+k)+k}-\mathrm{tau}\,(x1^2-i\,x2)}\,\sqrt{2\,\pi}}{(e^{\mathrm{tau}}\,(-1+k)-k)\,(e^{\mathrm{tau}}\,(-1+k)+k)\,\sqrt{1-2\,k+k^2-e^{-2\,\mathrm{tau}}\,k^2}}+$$
$$\frac{e^{\frac{(-1+e^{\mathrm{tau}})(-1+2k)\,x1^2}{e^{\mathrm{tau}}(-1+k)+k}-\mathrm{tau}\,(x1^2-i\,x2)}\,(-1+e^{\mathrm{tau}})^2\,\sqrt{2\,\pi}\,x1^2}{(e^{\mathrm{tau}}\,(-1+k)+k)^2\,\sqrt{1-2\,k+k^2-e^{-2\,\mathrm{tau}}\,k^2}}$$

Out[413]=
$$\frac{i\,e^{\frac{(-1+e^{\mathrm{tau}})(-1+2k)\,x1^2}{e^{\mathrm{tau}}(-1+k)+k}-\mathrm{tau}\,(x1^2-i\,x2)}\,(-1+e^{\mathrm{tau}})\,(-2\,e^{\mathrm{tau}}+e^{2\,\mathrm{tau}}\,(-1+k)-k)\,\sqrt{2\,\pi}\,x1}{(e^{\mathrm{tau}}\,(-1+k)-k)\,(e^{\mathrm{tau}}\,(-1+k)+k)^2\,\sqrt{1-2\,k+k^2-e^{-2\,\mathrm{tau}}\,k^2}}+$$
$$\frac{i\,e^{\frac{(-1+e^{\mathrm{tau}})(-1+2k)\,x1^2}{e^{\mathrm{tau}}(-1+k)+k}-\mathrm{tau}\,(x1^2-i\,x2)}\,(-1+e^{\mathrm{tau}})^3\,\sqrt{2\,\pi}\,x1^3}{(e^{\mathrm{tau}}\,(-1+k)+k)^3\,\sqrt{1-2\,k+k^2-e^{-2\,\mathrm{tau}}\,k^2}}$$

The current density

$$\tilde{J}^K(\vec{x},t) = \frac{e}{\sqrt{g}} \int d^3\theta \int d^3 J \, \delta(\vec{x} - \vec{x}_c(\theta, J)) \frac{\partial x_c^K}{\partial \theta^\alpha} \Omega^\alpha \tilde{f}(\vec{J},\vec{\theta},t)$$

$$\tilde{f}(\vec{J},\vec{\theta},t) = \sum_{\vec{m}} \tilde{f}_{\vec{m}}(\vec{J},t) e^{i\vec{m}\cdot\vec{\theta}}$$

$$\tilde{f}_{\vec{m}}(\vec{J},t) = e^{\frac{\alpha}{2}\frac{\bar{u}^2}{(1+2\alpha)}} \int_{-\infty}^{+\infty} d\bar{u}' \, G(\bar{u},\bar{u}') (1+2\alpha)^{-1} e^{-\frac{\alpha}{2}\frac{\bar{u}'^2}{(1+2\alpha)}} \left(-\frac{1}{\nu}\right) \tilde{Q}_{\vec{m}}(\bar{u}';t)$$

$$\bar{u} = \frac{u_\parallel - V_\parallel}{V_T} \sqrt{1+2\alpha} \quad , \quad \gamma = \frac{\alpha}{1+2\alpha} .$$

$$\tilde{Q}_{\vec{m}}(\bar{u}';t) = -\frac{e}{\omega} (\tilde{\mathcal{E}}_\alpha)_{\vec{m}} \left[\left(\vec{m}\cdot\frac{\partial f_0}{\partial \vec{J}}\right)\Omega^\alpha - (\vec{m}\cdot\vec{\Omega} - \omega)\frac{\partial f_0}{\partial J_\alpha} \right]_{\bar{u}'}$$

FLRE for $(\tilde{\mathcal{E}}_\alpha)_{\vec{m}}$:

$$\left(\tilde{\mathcal{E}}_\alpha^{(N)}\right)_{\vec{m}'} = \delta_{m_\vartheta k_\vartheta} \, \delta_{m_z k_z} \, e^{-i\omega t} \left(\sum_{h=0}^{N} [a_\alpha^j(h)]_e \frac{\partial^h}{\partial r_g^h} \right) \tilde{E}_j(r_g)$$

$$a_\alpha^j(h) = \left[\frac{\partial x_g^j}{\partial \theta^\alpha} + \frac{\partial \rho^j}{\partial \theta^\alpha} - \delta_{h0} \delta_r^j \frac{\partial \rho^r}{\partial \theta^\alpha} \right] \frac{(\rho^r)^h}{h!} e^{i\vec{k}\cdot\vec{\rho}}$$

$$\tilde{f}_{\vec{m}}(\vec{J},t) = \left(-\frac{1}{\nu}\right)(1+2\alpha)^{-1} e^{\frac{\gamma}{2}\bar{u}^2} \int_{-\infty}^{+\infty} d\bar{u}' \, G(\bar{u},\bar{u}') e^{-\frac{\gamma}{2}\bar{u}'^2} \left(-\frac{e}{\omega}\right) \times$$

$$\times \left[\left(\vec{m}\cdot\frac{\partial f_0}{\partial \vec{J}}\right)\Omega^\alpha - (\vec{m}\cdot\vec{\Omega} - \omega)\frac{\partial f_0}{\partial J_\alpha} \right]_{\bar{u}'} \delta_{m_\vartheta k_\vartheta} \, \delta_{m_z k_z} \, e^{-i\omega t} \left(\sum_{h=0}^{N} [a_\alpha^j(h)]_e \frac{\partial^h}{\partial r_g^h} \right) \tilde{E}_j$$

$$= \frac{ie}{\omega} \delta_{m_\vartheta k_\vartheta} \, \delta_{m_z k_z} \, e^{-i\omega t} \left(\sum_{h=0}^{N} [a_\alpha^j(h)]_e \frac{\partial^h}{\partial r_g^h} \right) \tilde{E}_j(r_g) \, f_{\vec{m}}^\alpha$$

$$f_{\vec{m}}^\alpha = i\left(-\frac{1}{\nu}\right)(1+2\alpha)^{-1} e^{\frac{\gamma}{2}\bar{u}^2} \int_{-\infty}^{+\infty} d\bar{u}' \, G(\bar{u},\bar{u}') e^{-\frac{\gamma}{2}\bar{u}'^2} \left[\left(\vec{m}\cdot\frac{\partial f_0}{\partial \vec{J}}\right)\Omega^\alpha - (\vec{m}\cdot\vec{\Omega} - \omega)\frac{\partial f_0}{\partial J_\alpha} \right]_{\bar{u}'}$$

$$\tilde{f}^{(N)}(\vec{J},\vec{\theta},t) = \sum_{\ell} e^{i\ell\phi + ik_\vartheta \vartheta_g + ik_z z_g} f^{(N)}_{\ell k_\vartheta k_z}(\vec{J},t)$$

$$\tilde{f}^{(N)}_{\vec{m}}(\vec{J},t) = \frac{ie}{\omega}\delta_{m_\vartheta k_\vartheta}\delta_{m_z k_z} e^{-i\omega t}\left(\sum_{n=0}^{N}[a^{j}_{\beta}(n)]_\ell \frac{\partial^n}{\partial r_g^n}\right)\tilde{E}_j(r_g) f^{\beta}_{\vec{m}}$$

$$f^{\beta}_{\vec{m}} = i\left(-\frac{1}{\nu}\right)(1+2\alpha)^{-1} e^{\frac{\gamma}{2}\bar{u}^2}\int_{-\infty}^{+\infty} d\bar{u}'\, G(\bar{u},\bar{u}')\, e^{-\frac{\gamma}{2}\bar{u}'^2} \times$$

$$\times \left[\left(\vec{m}\cdot\frac{\partial f_0}{\partial \vec{J}}\right)\Omega^{\beta} - (\vec{m}\cdot\vec{\Omega}-\omega)\frac{\partial f_0}{\partial J_\beta}\right]_{\bar{u}'}$$

$$\cancel{j^{K}_{(N)} = \sum_{n,n'=0}^{N}(-)^n \frac{\partial^n}{\partial r^n}\left(\sigma^{Kj}_{nn'}(r,\vec{K})\frac{\partial^{n'}}{\partial r^{n'}}\tilde{E}_j\right)}$$

$$\tilde{j}^{K}(\vec{x}) = \frac{2\pi i e^2}{\omega r}\sum_{n=0}^{N}\sum_{n'=0}^{N}(-)^n\frac{\partial^n}{\partial r^n}\left[\sigma^{Kj}_{nn'}\frac{\partial^{n'}}{\partial r^{n'}}\tilde{E}_j\right]$$

$$\sigma^{Kj}_{nn'}(r,\vec{K}) = \sum_{\ell}\int dJ_\perp\, du_\parallel \frac{\partial(P_\vartheta,P_z)}{\partial(r_0,u_\parallel)}[a^{K}_{\alpha}(n)]^{*}_{\ell}\,\Omega^{\alpha}[a^{j}_{\beta}(n')]_\ell f^{\beta}_{\vec{m}}$$

$$\frac{\partial(P_\vartheta,P_z)}{\partial(r_0,u_\parallel)} = m_0^2 r_0 \omega_c$$

$$f_0(\lambda,r_0,\bar{u}) = \frac{1}{(2\pi)^{3/2}}\frac{n_0}{m_0^3 V_T^3} e^{-\frac{1}{2}\lambda^2} e^{-\frac{1}{2}\bar{u}^2 s^2}$$

Factors

$$\frac{u_\parallel - V_\parallel}{v_T} = \bar u\,(1+2\alpha)^{-1/2} = \bar u s, \qquad u_\parallel = \bar u v_T s + V_\parallel .$$

$$\mathcal{J}_\perp = \frac{1}{2} m_0 \omega_c \rho^2 = \frac{1}{2}\frac{m_0 v_T^2}{\omega_c}\lambda^2, \qquad \lambda \equiv \frac{\rho}{\rho_L} = \frac{\omega_c}{v_T}\rho .$$

$$\Omega^\phi = \omega_c$$

$$\Omega^\vartheta = h^\vartheta(\bar u v_T s + V_\parallel) + V_E^\vartheta - \frac{h_z}{\omega_c}\left(h^\vartheta(\bar u v_T s + V_\parallel)^2 + V_E^\vartheta\right)^2 +$$

$$+ \frac{\hat h_z \omega_c'}{m_0 \omega_c r_0}\,\frac{1}{2}\frac{m_0 v_T^2}{\omega_c}\lambda^2 .$$

$$\Omega^z = h^z(\bar u v_T s + V_\parallel) + V_E^z + \frac{h^\vartheta r_0^2}{\omega_c}\left(h^\vartheta(\bar u v_T s + V_\parallel) + V_E^\vartheta\right)^2 -$$

$$- \frac{h^\vartheta r_0 \omega_c'}{m_0 \omega_c}\cdot\frac{1}{2}\frac{m_0 v_T^2}{\omega_c}\lambda^2 .$$

$$\begin{aligned}\Omega^\vartheta = {} & (V_E^\vartheta + V_\parallel^\vartheta)\left(1 - \frac{h_z}{\omega_c}(V_E^\vartheta + V_\parallel^\vartheta)\right) + \\ & h^\vartheta v_T s\left(1 - \frac{2h_z}{\omega_c}(V_E^\vartheta + V_\parallel^\vartheta)\right)\bar u + \\ & - \frac{(h^\vartheta)^2 h_z v_T^2 s^2}{\omega_c}\,\bar u^2 + \\ & \frac{h_z v_T^2 \omega_c'}{2\omega_c^2 r_0}\lambda^2 = Q_{00}^\vartheta + Q_{10}^\vartheta \bar u + Q_{20}^\vartheta \bar u^2 + Q_{02}^\vartheta \lambda^2 .\end{aligned}$$

$$\begin{aligned}\Omega^z = {} & V_E^z + V_\parallel^z + \frac{h^\vartheta r_0^2}{\omega_c}(V_E^\vartheta + V_\parallel^\vartheta)^2 + \\ & v_T s\left(h_z + \frac{2(h^\vartheta)^2 r_0^2}{\omega_c}(V_E^\vartheta + V_\parallel^\vartheta)\right)\bar u + \\ & \frac{(h^\vartheta)^3 r_0^2 s^2 v_T^2}{\omega_c}\,\bar u^2 + \\ & - \frac{h^\vartheta r_0 v_T^2 \omega_c'}{2\omega_c^2}\lambda^2 = Q_{00}^z + Q_{10}^z \bar u + Q_{20}^z \bar u^2 + Q_{02}^z \lambda^2 .\end{aligned}$$

$$\frac{\partial P_\vartheta}{\partial u_\parallel} = m_0 h_\vartheta - \frac{m_0 h_z}{\omega_c} 2\left(h^\vartheta u_\parallel + V_E^\vartheta\right) h^\vartheta r_0^2 = m_0 h_\vartheta - \frac{2 m_0 h_z h^\vartheta r_0^2}{\omega_c} V_E^\vartheta$$

$$- \frac{2 m_0 h_z h^\vartheta r_0^2}{\omega_c} h^\vartheta u_\parallel = m_0 h_\vartheta - \frac{2 m_0 h_z h^\vartheta r_0^2}{\omega_c} V_E^\vartheta -$$

$$- \frac{2 m_0 h_z (h^\vartheta)^2 r_0^2}{\omega_c}\left(v_T s \bar{u} + V_\parallel\right) = m_0 h_\vartheta - \frac{2 m_0 h_z h^\vartheta r_0^2}{\omega_c}\left(V_E^\vartheta + V_\parallel^\vartheta\right) -$$

$$- \frac{2 m_0 h_z (h^\vartheta)^2 r_0^2 v_T s}{\omega_c} \bar{u}.$$

$$\frac{\partial P_z}{\partial u_\parallel} = m_0 h_z + \frac{m_0 h^\vartheta}{\omega_c} r_0^2 \left(h^\vartheta u_\parallel + V_E^\vartheta\right) \cdot 2 \cdot h^\vartheta = m_0 h_z + \frac{2 (h^\vartheta)^2 m_0 r_0^2}{\omega_c} V_E^\vartheta +$$

$$+ \frac{2 m_0 (h^\vartheta)^2 r_0^2}{\omega_c} h^\vartheta \left(v_T s \bar{u} + V_\parallel\right) = m_0 h_z + \frac{2 m_0 (h^\vartheta)^2 r_0^2}{\omega_c}\left(V_E^\vartheta + V_\parallel^\vartheta\right) +$$

$$+ \frac{2 m_0 (h^\vartheta)^3 r_0^2 v_T s}{\omega_c} \bar{u}.$$

$$\frac{\partial P_\vartheta}{\partial r_0} = \frac{e}{c} A'_{0\vartheta} + m_0 h'_\vartheta u_\parallel + m_0 V'_{E\vartheta} - \cancel{\frac{m_0 h_z}{\omega_c}(h^\vartheta)} - \left(\frac{m_0 h_z r_0^2}{\omega_c}\right)' \left(h^\vartheta u_\parallel + V_E^\vartheta\right)^2$$

$$- \frac{m_0 h_z r_0^2}{\omega_c} 2\left(h^\vartheta u_\parallel + V_E^\vartheta\right)\left((h^\vartheta)' u_\parallel + (V_E^\vartheta)'\right) = m_0 r_0 \omega_c h_z + m_0 h'_\vartheta u_\parallel +$$

$$+ m_0 V'_{E\vartheta} - \left(\frac{m_0 h_z r_0^2}{\omega_c}\right)' \left((h^\vartheta)^2 u_\parallel^2 + 2 h^\vartheta u_\parallel V_E^\vartheta + (V_E^\vartheta)^2\right) -$$

$$- \frac{m_0 h_z r_0^2}{\omega_c} 2 \left(h^\vartheta (h^\vartheta)' u_\parallel^2 + \left[V_E^\vartheta (h^\vartheta)' + h^\vartheta (V_E^\vartheta)'\right] u_\parallel + V_E^\vartheta (V_E^\vartheta)'\right) =$$

$$= m_0 r_0 \omega_c h_z + m_0 V'_{E\vartheta} - \left(\frac{m_0 h_z r_0^2}{\omega_c}\right)' (V_E^\vartheta)^2 - \frac{m_0 h_z r_0^2}{\omega_c} 2 V_E^\vartheta (V_E^\vartheta)' +$$

$$+ u_\parallel \left\{ m_0 h'_\vartheta - \left(\frac{m_0 h_z r_0^2}{\omega_c}\right)' 2 h^\vartheta V_E^\vartheta - \frac{m_0 h_z r_0^2}{\omega_c} 2\left[V_E^\vartheta (h^\vartheta)' + h^\vartheta (V_E^\vartheta)'\right] \right\}$$

$$- \left\{ \left(\frac{m_0 h_z r_0^2}{\omega_c}\right)' (h^\vartheta)^2 + \frac{m_0 h_z r_0^2}{\omega_c} 2 (h^\vartheta)(h^\vartheta)' \right\} u_\parallel^2 =$$

$$\frac{\partial P_\vartheta}{\partial u_\parallel} = m_0 h_\vartheta - \frac{m_0 h_z}{\omega_c} 2\left(h^\vartheta u_\parallel + V_E^\vartheta\right) h^\vartheta r_0^2 = m_0 h_\vartheta - \frac{2 m_0 h_z h^\vartheta r_0^2}{\omega_c} V_E^\vartheta$$

$$- \frac{2 m_0 h_z h^\vartheta r_0^2}{\omega_c} h^\vartheta u_\parallel = m_0 h_\vartheta - \frac{2 m_0 h_z h^\vartheta r_0^2}{\omega_c} V_E^\vartheta -$$

$$- \frac{2 m_0 h_z (h^\vartheta)^2 r_0^2}{\omega_c}\left(v_T s \bar{u} + V_\parallel\right) = m_0 h_\vartheta - \frac{2 m_0 h_z h^\vartheta r_0^2}{\omega_c}\left(V_E^\vartheta + V_\parallel^\vartheta\right) -$$

$$- \frac{2 m_0 h_z (h^\vartheta)^2 r_0^2 v_T s}{\omega_c} \bar{u}.$$

$$\frac{\partial P_z}{\partial u_\parallel} = m_0 h_z + \frac{m_0 h^\vartheta}{\omega_c} r_0^2 \left(h^\vartheta u_\parallel + V_E^\vartheta\right) \cdot 2 \cdot h^\vartheta = m_0 h_z + \frac{2 (h^\vartheta)^2 m_0 r_0^2}{\omega_c} V_E^\vartheta +$$

$$+ \frac{2 m_0 (h^\vartheta)^2 r_0^2}{\omega_c} h^\vartheta \left(v_T s \bar{u} + V_\parallel\right) = m_0 h_z + \frac{2 m_0 (h^\vartheta)^2 r_0^2}{\omega_c}\left(V_E^\vartheta + V_\parallel^\vartheta\right) +$$

$$+ \frac{2 m_0 (h^\vartheta)^3 r_0^2 v_T s}{\omega_c} \bar{u}.$$

$$\frac{\partial P_\vartheta}{\partial r_0} = \frac{e}{c} A'_{0\vartheta} + m_0 h'_\vartheta u_\parallel + m_0 V'_{E\vartheta} - \cancel{\frac{m_0 h_z}{\omega_c}} - \left(\frac{m_0 h_z r_0^2}{\omega_c}\right)' \left(h^\vartheta u_\parallel + V_E^\vartheta\right)^2$$

$$- \frac{m_0 h_z r_0^2}{\omega_c} 2\left(h^\vartheta u_\parallel + V_E^\vartheta\right)\left((h^\vartheta)' u_\parallel + (V_E^\vartheta)'\right) = m_0 r_0 \omega_c h_z + m_0 h'_\vartheta u_\parallel +$$

$$+ m_0 V'_{E\vartheta} - \left(\frac{m_0 h_z r_0^2}{\omega_c}\right)' \left((h^\vartheta)^2 u_\parallel^2 + 2 h^\vartheta u_\parallel V_E^\vartheta + (V_E^\vartheta)^2\right) -$$

$$- \frac{m_0 h_z r_0^2}{\omega_c} 2\left(h^\vartheta (h^\vartheta)' u_\parallel^2 + \left[V_E^\vartheta (h^\vartheta)' + h^\vartheta (V_E^\vartheta)'\right] u_\parallel + V_E^\vartheta (V_E^\vartheta)'\right) =$$

$$= m_0 r_0 \omega_c h_z + m_0 V'_{E\vartheta} - \left(\frac{m_0 h_z r_0^2}{\omega_c}\right)' (V_E^\vartheta)^2 - \frac{m_0 h_z r_0^2}{\omega_c} 2 V_E^\vartheta (V_E^\vartheta)' +$$

$$+ u_\parallel \left\{ m_0 h'_\vartheta - \left(\frac{m_0 h_z r_0^2}{\omega_c}\right)' 2 h^\vartheta V_E^\vartheta - \frac{m_0 h_z r_0^2}{\omega_c} 2\left[V_E^\vartheta (h^\vartheta)' + h^\vartheta (V_E^\vartheta)'\right]\right\}$$

$$- \left\{ \left(\frac{m_0 h_z r_0^2}{\omega_c}\right)' (h^\vartheta)^2 + \frac{m_0 h_z r_0^2}{\omega_c} 2 (h^\vartheta)(h^\vartheta)' \right\} u_\parallel^2 =$$

$$= m_0 r_0 \omega_c h_z + m_0 V'_{E\vartheta} - \left(\frac{m_0 h_z r_0^2 (V_E^\vartheta)^2}{\omega_c}\right)' + u_{\parallel}\left\{ m_0 h'_\vartheta - \right.$$

$$\left. - \left(\frac{m_0 h_z r_0^2 h^\vartheta V_E^\vartheta}{\omega_c}\right)' \cdot 2 \right\} - u_{\parallel}^2 \left\{ \left(\frac{m_0 h_z r_0^2 (h^\vartheta)^2}{\omega_c}\right)' \right\} =$$

$$= m_0 r_0 \omega_c h_z + m_0 V'_{E\vartheta} - \left(\frac{m_0 h_z r_0^2 (V_E^\vartheta)^2}{\omega_c}\right)' + \left\{ m_0 h'_\vartheta - \left(\frac{m_0 h_z r_0^2 h^\vartheta V_E^\vartheta}{\omega_c}\right)' 2 \right\}$$

$$\times \left(v_T s \bar{u} + V_{\parallel} \right) - \left(v_T^2 s^2 \bar{u}^2 + 2 V_{\parallel} v_T s \bar{u} + V_{\parallel}^2 \right) \left(\frac{m_0 h_z r_0^2 (h^\vartheta)^2}{\omega_c}\right)' =$$

$$= m_0 r_0 \omega_c h_z + m_0 V'_{E\vartheta} - \left(\frac{m_0 h_z r_0^2 (V_E^\vartheta)^2}{\omega_c}\right)' + m_0 h'_\vartheta V_{\parallel} -$$

$$- 2 V_{\parallel} \left(\frac{m_0 h_z r_0^2 h^\vartheta V_E^\vartheta}{\omega_c}\right)' - \left(\frac{m_0 h_z r_0^2 (h^\vartheta)^2}{\omega_c}\right)' V_{\parallel}^2 +$$

$$+ \bar{u} \left\{ \left\{ m_0 h'_\vartheta - 2\left(\frac{m_0 h_z r_0^2 h^\vartheta V_E^\vartheta}{\omega_c}\right)' \right\} v_T s - \left(\frac{m_0 h_z r_0^2 (h^\vartheta)^2}{\omega_c}\right)' v_T s \, 2 V_{\parallel} \right\}$$

$$+ \bar{u}^2 \left\{ - v_T^2 s^2 \left(\frac{m_0 h_z r_0^2 (h^\vartheta)^2}{\omega_c}\right)' \right\} = m_0 r_0 \omega_c h_z + m_0 V'_{E\vartheta} + m_0 h'_\vartheta V_{\parallel} -$$

$$- \left(\frac{m_0 h_z r_0^2}{\omega_c} \left(V_E^\vartheta + V_{\parallel}^\vartheta\right)^2 \right)' + \frac{2 m_0 h_z r_0^2}{\omega_c} h^\vartheta V'_{\parallel} \left(V_E^\vartheta + V_{\parallel}^\vartheta\right) +$$

$$+ \bar{u} \left[v_T s \, m_0 h'_\vartheta - 2 v_T s \left(\left(\frac{m_0 h_z r_0^2 h^\vartheta}{\omega_c} \left(V_E^\vartheta + V_{\parallel}^\vartheta\right)\right)' - \frac{m_0 h_z r_0^2 h^\vartheta}{\omega_c} h^\vartheta V'_{\parallel} \right) \right] +$$

$$+ \bar{u}^2 \left\{ - v_T^2 s^2 \left(\frac{m_0 h_z r_0^2 (h^\vartheta)^2}{\omega_c}\right)' \right\}$$

$$\frac{\partial P_z}{\partial r_0} = \frac{e}{c} A'_{0z} + m_0 h'_z u_{\parallel} + m_0 V'_{Ez} + \left(\frac{m_0 h^\vartheta r_0}{\omega_c}\right)' \left(h^\vartheta u_{\parallel} + V_E^\vartheta\right)^2 +$$

$$+ \frac{m_0 h^\vartheta r_0^2}{\omega_c} \cdot 2 \left(h^\vartheta u_{\parallel} + V_E^\vartheta\right) \left((h^\vartheta)' u_{\parallel} + (V_E^\vartheta)' \right) = - m_0 \omega_c \hat{h}_\vartheta +$$

$$+ m_0 h'_z u_{\parallel} + m_0 V'_{Ez} + \left(\frac{m_0 h^\vartheta r_0^2}{\omega_c}\right)' \left((h^\vartheta)^2 u_{\parallel}^2 + 2 h^\vartheta V_E^\vartheta u_{\parallel} + (V_E^\vartheta)^2 \right) +$$

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$$+\frac{2m_0 h^\vartheta r_0^2}{\omega_c}\left[h^\vartheta (h^\vartheta)' u_\parallel^2 + \left(h^\vartheta (V_E^\vartheta)' + V_E^\vartheta (h^\vartheta)'\right)u_\parallel + V_E^\vartheta (V_E^\vartheta)'\right] =$$

$$= -m_0\omega_c h^\vartheta r_0 + m_0 V'_{Ez} + \left(\frac{m_0 h^\vartheta r_0^2}{\omega_c}\right)'(V_E^\vartheta)^2 + \frac{2m_0 h^\vartheta r_0^2}{\omega_c} V_E^\vartheta (V_E^\vartheta)' +$$

$$+ u_\parallel\left[m_0 h'_z + \left(\frac{m_0 h^\vartheta r_0^2}{\omega_c}\right)' 2h^\vartheta V_E^\vartheta + \frac{2m_0 h^\vartheta r_0^2}{\omega_c}\left(h^\vartheta V_E^\vartheta\right)'\right] +$$

$$+ u_\parallel^2\left[\left(\frac{m_0 h^\vartheta r_0^2}{\omega_c}\right)'(h^\vartheta)^2 + \frac{2m_0 h^\vartheta r_0^2}{\omega_c} h^\vartheta (h^\vartheta)'\right] =$$

$$= -m_0\omega_c r_0 h^\vartheta + m_0 V'_{Ez} + \left(\frac{m_0 h^\vartheta r_0^2 (V_E^\vartheta)^2}{\omega_c}\right)' +$$

$$+ u_\parallel\left[m_0 h'_z + 2\left(\frac{m_0 h^\vartheta r_0^2 h^\vartheta V_E^\vartheta}{\omega_c}\right)'\right] + u_\parallel^2\left(\frac{m_0 h^\vartheta r_0^2 (h^\vartheta)^2}{\omega_c}\right)'.$$

$$\frac{\partial P_z}{\partial r_0} = -m_0\omega_c r_0 h^\vartheta + m_0 V'_{Ez} + \left(\frac{m_0 h^\vartheta r_0^2 (V_E^\vartheta)^2}{\omega_c}\right)' +$$

$$+ \left(v_T s\bar u + V_\parallel\right)\left[\quad\right] + \left(v_T^2 s^2 \bar u^2 + 2v_T s V_\parallel \bar u + V_\parallel^2\right)\left(\frac{m_0 h^\vartheta r_0^2 (h^\vartheta)^2}{\omega_c}\right)' =$$

$$= -m_0\omega_c r_0 h^\vartheta + m_0 V'_{Ez} + \left(\frac{m_0 h^\vartheta r_0^2 (V_E^\vartheta)^2}{\omega_c}\right)' + V_\parallel m_0 h'_z +$$

$$+ 2V_\parallel\left(\frac{m_0 h^\vartheta r_0^2 h^\vartheta V_E^\vartheta}{\omega_c}\right)' + V_\parallel^2\left(\frac{m_0 h^\vartheta r_0^2 (h^\vartheta)^2}{\omega_c}\right)' +$$

$$+ \bar u\left[v_T s m_0 h'_z + 2v_T s\left(\frac{m_0 h^\vartheta r_0^2 h^\vartheta V_E^\vartheta}{\omega_c}\right)' + 2v_T s V_\parallel\left(\frac{m_0 h^\vartheta r_0^2 (h^\vartheta)^2}{\omega_c}\right)'\right]$$

$$+ \bar u^2\left[v_T^2 s^2\left(\frac{m_0 h^\vartheta r_0^2 (h^\vartheta)^2}{\omega_c}\right)'\right] =$$

$$= -m_0\omega_c r_0 h^{\vartheta} + m_0 V_{Ez}' + m_0 h_z' V_{\parallel} + \left(\frac{m_0 h^{\vartheta} r_0^2}{\omega_c}\left(V_E^{\vartheta}+V_{\parallel}^{\vartheta}\right)^2\right)' -$$

$$- 2\frac{m_0 (h^{\vartheta})^2 r_0^2}{\omega_c} V_{\parallel}'\left(V_E^{\vartheta}+V_{\parallel}^{\vartheta}\right) + \bar{u}\left[V_T s\, m_0 h_z' + 2V_T s\left(\frac{m_0 (h^{\vartheta})^2 r_0^2}{\omega_c}\times\right.\right.$$

$$\left.\left.\times\left(V_E^{\vartheta}+V_{\parallel}^{\vartheta}\right)\right)' - 2V_T s\,\frac{m_0 (h^{\vartheta})^2 r_0^2}{\omega_c} h^{\vartheta} V_{\parallel}'\right] + \bar{u}^2\left[V_T^2 s^2\left(\frac{m_0 h^{\vartheta} r_0^2 (h^{\vartheta})^2}{\omega_c}\right)'\right].$$

$$\frac{\partial f_0}{\partial \mathcal{I}_\perp} = \left(-\frac{f_0}{T}\right)\omega_c$$

$$\frac{\partial f_0}{\partial P_{\vartheta}} = \frac{\partial(r_0,u_{\parallel})}{\partial(P_{\vartheta},P_z)}\left[\frac{\partial f_0}{\partial r_0}\frac{\partial P_z}{\partial u_{\parallel}} - \frac{\partial f_0}{\partial u_{\parallel}}\frac{\partial P_z}{\partial r_0}\right]$$

$$\frac{\partial f_0}{\partial P_z} = \frac{\partial(r_0,u_{\parallel})}{\partial(P_{\vartheta},P_z)}\left[-\frac{\partial f_0}{\partial r_0}\frac{\partial P_{\vartheta}}{\partial u_{\parallel}} + \frac{\partial f_0}{\partial u_{\parallel}}\frac{\partial P_{\vartheta}}{\partial r_0}\right]$$

$$\frac{\partial f_0}{\partial r_0} = \left(-\frac{f_0}{T}\right)\left\{-\frac{h'}{n}T + \frac{3}{2}T' + \mathcal{I}_\perp\left(\frac{\omega_c}{T}\right)'T - m_0 V_{\parallel}'(u_{\parallel}-V_{\parallel}) - \frac{m_0}{2}\frac{T'}{T}(u_{\parallel}-V_{\parallel})^2\right\} =$$

$$= \left(-\frac{f_0}{T}\right)\left\{-\frac{h'}{n}T + \frac{3}{2}T' + T\left(\frac{\omega_c}{T}\right)'\cdot\frac{1}{2}\frac{m_0 V_T^2}{\omega_c}\lambda^2 - m_0 V_{\parallel}'\bar{u}V_T s -\right.$$

$$\left.-\frac{m_0}{2}\frac{T'}{T}\bar{u}^2 V_T^2 s^2\right\} = \left(-\frac{f_0}{T}\right)\left\{-\frac{h'}{n}m_0 V_T^2 + \frac{3}{\cancel{2}}m_0\cancel{2}V_T V_T' +\right.$$

$$\left.+ m_0 V_T^2\left(\frac{\omega_c}{m_0 V_T^2}\right)'\cdot\frac{1}{2}\frac{m_0 V_T^2}{\omega_c}\lambda^2 - m_0 V_T s V_{\parallel}'\bar{u} - \frac{\cancel{m_0}}{\cancel{2}}\frac{m_0\cancel{2}V_T V_T'}{\cancel{m_0 V_T^2}}\cancel{V_T}s^2\bar{u}^2\right\} =$$

$$= \left(-\frac{f_0}{T}\right)m_0 V_T^2\left\{-\frac{h'}{n} + 3\frac{V_T'}{V_T} + \frac{1}{2}\left(\frac{\omega_c}{m_0 V_T^2}\right)'\left(\frac{m_0 V_T^2}{\omega_c}\right)\lambda^2 - \frac{sV_{\parallel}'}{V_T}\bar{u} -\right.$$

$$\left.-\frac{V_T'}{V_T}s^2\bar{u}^2\right\}$$

$$\frac{\partial f_0}{\partial r_0} = \left(-\frac{f_0}{T}\right)m_0 V_T^2\left\{-\frac{h'}{n} + 3\frac{V_T'}{V_T} + \frac{1}{2}\left(\frac{\omega_c}{m_0 V_T^2}\right)'\left(\frac{m_0 V_T^2}{\omega_c}\right)\lambda^2 - \frac{sV_{\parallel}'}{V_T}\bar{u} - \frac{V_T'}{V_T}s^2\bar{u}^2\right\}$$

$$\frac{\partial f_0}{\partial q_{\parallel}} = \left(-\frac{f_0}{T}\right) m_0 \bar{u} V_T s = \left(-\frac{f_0}{T}\right) m_0 V_T^2 \frac{s}{V_T}\bar{u}$$

$$\frac{\partial f_0}{\partial \rho_s} = \gamma^{-1}\left\{\left(-\frac{f_0}{T}\right) m_0 V_T^2\right\}\left\{\left(-\frac{n'}{n} + 3\frac{V_T'}{V_T} + \frac{1}{2}\frac{(\ )'}{(\ )}\lambda^2 - \frac{sV_{\parallel}'}{V_T}\bar{u} - \frac{V_T'}{V_T}s^2\bar{u}^2\right)\times\right.$$

$$\times\left(m_0 h_z + \frac{2m_0(h^{\vartheta})^2 r_0^2}{\omega_c}(V_E^{\vartheta}+V_{\parallel}^{\vartheta}) + \frac{2m_0(h^{\vartheta})^3 r_0^2 V_T s}{\omega_c}\bar{u}\right) -$$

$$-\frac{s}{V_T}\bar{u}\left(-m_0\omega_c r_0 h^{\vartheta} + m_0 V_{Ez}' + m_0 h_z' V_{\parallel} + \left(\frac{m_0 h^{\vartheta} r_0^2}{\omega_c}(V_E^{\vartheta}+V_{\parallel}^{\vartheta})^2\right)' -\right.$$

$$-2\frac{m_0(h^{\vartheta})^2 r_0^2}{\omega_c}V_{\parallel}'(V_E^{\vartheta}+V_{\parallel}^{\vartheta}) + \bar{u}\left[V_T s m_0 h_z' + 2V_T s\left(\frac{m_0(h^{\vartheta})^2 r_0^2}{\omega_c}(V_E^{\vartheta}+V_{\parallel}^{\vartheta})\right)'\right.$$

$$\left.\left.\left.-2V_T s\frac{m_0(h^{\vartheta})^2 r_0^2}{\omega_c}h^{\vartheta}V_{\parallel}'\right] + \bar{u}^2\left[V_T^2 s^2\left(\frac{m_0 h^{\vartheta} r_0^2 (h^{\vartheta})^2}{\omega_c}\right)'\right]\right)\right\} =$$

$$= \gamma^{-1}\left(-\frac{f_0}{T}\right) m_0 V_T^2\left\{\frac{1}{2}\left(\frac{\omega_c}{m_0 V_T^2}\right)'\left(\frac{m_0 V_T^2}{\omega_c}\right)\left(m_0 h_z + \frac{2m_0(h^{\vartheta})^2 r_0^2}{\omega_c}(V_E^{\vartheta}+V_{\parallel}^{\vartheta}) +\right.\right.$$

$$\left.+\frac{2m_0(h^{\vartheta})^3 r_0^2 V_T s}{\omega_c}\bar{u}\right)\lambda^2 + \bar{u}^0\left[\left(-\frac{n'}{n}+3\frac{V_T'}{V_T}\right)\left(m_0 h_z + \frac{2m_0(h^{\vartheta})^2 r_0^2}{\omega_c}(V_E^{\vartheta}+V_{\parallel}^{\vartheta})\right)\right]$$

$$+\bar{u}^1\left[\left(-\frac{n'}{n}+3\frac{V_T'}{V_T}\right)\cdot\frac{2m_0(h^{\vartheta})^3 r_0^2 V_T s}{\omega_c} - \frac{sV_{\parallel}'}{V_T}\cdot\left(m_0 h_z + \frac{2m_0(h^{\vartheta})^2 r_0^2}{\omega_c}(V_E^{\vartheta}+V_{\parallel}^{\vartheta})\right)+\right.$$

$$+\frac{s}{V_T}m_0\omega_c r_0 h^{\vartheta} - \frac{s}{V_T}m_0\left(V_{Ez}' + h_z' V_{\parallel}\right) - \frac{s}{V_T}\left(\frac{m_0 h^{\vartheta} r_0^2}{\omega_c}(V_E^{\vartheta}+V_{\parallel}^{\vartheta})^2\right)' +$$

$$\left.+\frac{s}{V_T}2\frac{m_0(h^{\vartheta})^2 r_0^2}{\omega_c}(V_E^{\vartheta}+V_{\parallel}^{\vartheta})V_{\parallel}'\right] + \bar{u}^2\left[-\frac{sV_{\parallel}'}{V_T}\frac{2m_0(h^{\vartheta})^3 r_0^2 V_T s}{\omega_c} -\right.$$

$$-\frac{V_T'}{V_T}s^2\left(m_0 h_z + \frac{2m_0(h^{\vartheta})^2 r_0^2}{\omega_c}(V_E^{\vartheta}+V_{\parallel}^{\vartheta})\right) - \frac{s}{V_T}\left(V_T s m_0 h_z' +\right.$$

$$\left.\left.+2V_T s\left(\frac{m_0(h^{\vartheta})^2 r_0^2}{\omega_c}(V_E^{\vartheta}+V_{\parallel}^{\vartheta})\right)' - 2V_T s\frac{m_0(h^{\vartheta})^2 r_0^2}{\omega_c}h^{\vartheta}V_{\parallel}'\right)\right] +$$

$$+\bar{u}^3\left[-\frac{V_T'}{V_T}s^2\frac{2m_0(h^{\vartheta})^3 r_0^2 V_T s}{\omega_c} - \frac{s}{V_T}V_T^2 s^2\left(\frac{m_0 h^{\vartheta} r_0^2 (h^{\vartheta})^2}{\omega_c}\right)'\right] =$$

$$
\begin{aligned}
\frac{\partial f_0}{\partial P_\vartheta} &= y^{-1}\Big(-\frac{f_0}{T}\Big) m_0 V_T^2 \bigg\{ \frac{1}{2}\Big(\frac{\omega_c}{m_0 V_T^2}\Big)' \Big(\frac{m_0 V_T^2}{\omega_c}\Big) \bigg(m_0 h_z + \frac{2m_0 (h^\vartheta)^2 r_0^2}{\omega_c}\big(V_E^\vartheta + V_\parallel^\vartheta\big) + \\
&+ \frac{2m_0 (h^\vartheta)^3 r_0^2 V_T s}{\omega_c}\bar{u} \bigg)\lambda^2 + \bar{u}^0 \bigg[\Big(-\frac{h'}{h} + 3\frac{V_T'}{V_T}\Big)\Big(m_0 h_z + \frac{2m_0 (h^\vartheta)^2 r_0^2}{\omega_c}\big(V_E^\vartheta + V_\parallel^\vartheta\big)\Big)\bigg] \\
&+ \bar{u}^1 \bigg[\Big(-\frac{h'}{h} + 3\frac{V_T'}{V_T}\Big) \frac{2m_0 r_0^2 V_T (h^\vartheta)^3 s}{\omega_c} + \frac{s}{V_T} m_0 \Big[\omega_c r_0 h^\vartheta - \big(V_{Ez} + h_z V_\parallel\big)'\Big] + \\
&- \frac{s}{V_T}\Big(\frac{m_0 h^\vartheta r_0^2}{\omega_c}\big(V_E^\vartheta + V_\parallel^\vartheta\big)^2\Big)' \bigg] + \bar{u}^2 \bigg[-\frac{s}{V_T} V_T s m_0 h_z' - \\
&- \frac{V_T'}{V_T} s^2 \Big(m_0 h_z + \frac{2m_0 (h^\vartheta)^2 r_0^2}{\omega_c}\big(V_E^\vartheta + V_\parallel^\vartheta\big)\Big) - \frac{s}{V_T} 2V_T s \Big(\frac{m_0 (h^\vartheta)^2 r_0^2}{\omega_c}\big(V_E^\vartheta + V_\parallel^\vartheta\big)\Big)' \bigg] \\
&+ \bar{u}^3 \bigg[-\frac{s^3}{V_T}\Big(\frac{m_0 h^\vartheta r_0^2 (h^\vartheta)^2 V_T^2}{\omega_c}\Big)' \bigg] = y^{-1}\Big(-\frac{f_0}{T}\Big) m_0 V_T^2 \bigg\{ \frac{1}{2}\Big(\frac{\omega_c}{m_0 V_T^2}\Big)' \Big(\frac{m_0 V_T^2}{\omega_c}\Big) \times \\
&\bigg[m_0 h_z + \frac{2m_0 (h^\vartheta)^2 r_0^2}{\omega_c}\big(V_E^\vartheta + V_\parallel^\vartheta\big) + \frac{2m_0 (h^\vartheta)^3 r_0^2 V_T s}{\omega_c}\bar{u} \bigg]\lambda^2 + \\
&+ \bar{u}^0 \bigg[\Big(-\frac{h'}{h} + 3\frac{V_T'}{V_T}\Big)\Big(m_0 h_z + \frac{2m_0 (h^\vartheta)^2 r_0^2}{\omega_c}\big(V_E^\vartheta + V_\parallel^\vartheta\big)\Big)\bigg] + \\
&+ \bar{u}^1 \bigg[\Big(-\frac{h'}{h} + 3\frac{V_T'}{V_T}\Big) \frac{2m_0 r_0^2 V_T (h^\vartheta)^3 s}{\omega_c} + \frac{s}{V_T} m_0 \Big[\omega_c r_0 h^\vartheta - \big(V_{Ez} + h_z V_\parallel\big)'\Big] - \\
&- \frac{s}{V_T}\Big(\frac{m_0 h^\vartheta r_0^2}{\omega_c}\big(V_E^\vartheta + V_\parallel^\vartheta\big)^2\Big)' \bigg] + \bar{u}^2 \bigg[-\frac{s^2}{V_T}\big(h_z V_T\big)' m_0 - \\
&- \frac{s^2}{V_T} 2 \Big(\frac{m_0 (h^\vartheta)^2 r_0^2 V_T}{\omega_c}\big(V_E^\vartheta + V_\parallel^\vartheta\big)\Big)' \bigg] + \bar{u}^3 \bigg[-\frac{s^3}{V_T}\Big(\frac{m_0 h^\vartheta r_0^2 (h^\vartheta)^2 V_T^2}{\omega_c}\Big)' \bigg].
\end{aligned}
$$

$$\frac{\partial f_0}{\partial P_z} = \gamma^{-1}\left(-\frac{f_0}{T}\right) m_0 V_T^2 \left\{ \left(\frac{h'}{h} - 3\frac{V_T'}{V_T} - \frac{1}{2}\left(\frac{\omega_c}{m_0 V_T^2}\right)'\left(\frac{m_0 V_T^2}{\omega_c}\right)\right)\lambda^2 + \right.$$

$$+ \frac{s V_\parallel'}{V_T}\bar{u} + \frac{V_T'}{V_T}s^2\bar{u}^2 \Big) \left(m_0 h_\vartheta - \frac{2 m_0 h_z h^\vartheta r_0^2}{\omega_c}\left(V_E^\vartheta + V_\parallel^\vartheta\right) - \frac{2 m_0 h_z (h^\vartheta)^2 r_0^2 V_T s}{\omega_c}\bar{u}\right)$$

$$+ \frac{s}{V_T}\bar{u}\left(m_0 r_0 \omega_c h_z + m_0 V_{E\vartheta}' + m_0 h_\vartheta' V_\parallel - \left(\frac{m_0 h_z r_0^2}{\omega_c}\left(V_E^\vartheta + V_\parallel^\vartheta\right)^2\right)' + \right.$$

$$+ \frac{2 m_0 h_z r_0^2}{\omega_c} h^\vartheta V_\parallel'\left(V_E^\vartheta + V_\parallel^\vartheta\right) + \bar{u}\left[V_T s\, m_0 h_\vartheta' - 2 V_T s\left(\left(\frac{m_0 h_z r_0^2 h^\vartheta}{\omega_c}\left(V_E^\vartheta + V_\parallel^\vartheta\right)\right)'\right.\right.$$

$$\left.\left.\left. - \frac{m_0 h_z r_0^2 h^\vartheta}{\omega_c} h^\vartheta V_\parallel'\right)\right] + \bar{u}^2\left[- V_T^2 s^2 \left(\frac{m_0 h_z r_0^2 (h^\vartheta)^2}{\omega_c}\right)'\right]\right) =$$

$$= \gamma^{-1}\left(-\frac{f_0}{T}\right) m_0 V_T^2 \left\{ -\frac{1}{2}\left(\frac{\omega_c}{m_0 V_T^2}\right)'\left(\frac{m_0 V_T^2}{\omega_c}\right)\left[m_0 h_\vartheta - \frac{2 m_0 h_z h^\vartheta r_0^2}{\omega_c}\left(V_E^\vartheta + V_\parallel^\vartheta\right) - \right.\right.$$

$$\left. - \frac{2 m_0 h_z (h^\vartheta)^2 r_0^2 V_T s}{\omega_c}\bar{u}\right]\lambda^2 + \bar{u}^0\left[\left(\frac{h'}{h} - 3\frac{V_T'}{V_T}\right)\left(m_0 h_\vartheta - \frac{2 m_0 h_z h^\vartheta r_0^2}{\omega_c}\left(V_E^\vartheta + V_\parallel^\vartheta\right)\right)\right]$$

$$+ \bar{u}^1\left[\left(\frac{h'}{h} - 3\frac{V_T'}{V_T}\right)(-)\frac{2 m_0 h_z (h^\vartheta)^2 r_0^2 V_T s}{\omega_c} + \frac{s V_\parallel'}{V_T}\left(\left(m_0 h_\vartheta\right) - \cancel{\frac{2 m_0 h_z h^\vartheta r_0^2}{\omega_c}} \times \right.\right.$$

$$\left.\cancel{\left(V_E^\vartheta + V_\parallel^\vartheta\right)}\right) + \frac{s}{V_T}\left(m_0 r_0 \omega_c h_z + m_0 V_{E\vartheta}' + m_0 h_\vartheta' V_\parallel - \left(\frac{m_0 h_z r_0^2}{\omega_c}\left(V_E^\vartheta + V_\parallel^\vartheta\right)^2\right)' + \right.$$

$$\left.\left. + \cancel{\frac{2 m_0 h_z r_0^2}{\omega_c} h^\vartheta V_\parallel'\left(V_E^\vartheta + V_\parallel^\vartheta\right)}\right)\right] + \bar{u}^2\left[\frac{s V_\parallel'}{V_T}(-)\cancel{\frac{2 m_0 h_z (h^\vartheta)^2 r_0^2 V_T s}{\omega_c}} + \right.$$

$$+ \frac{V_T'}{V_T}s^2\left(m_0 h_\vartheta - \frac{2 m_0 h_z h^\vartheta r_0^2}{\omega_c}\left(V_E^\vartheta + V_\parallel^\vartheta\right)\right) + \frac{s}{V_T}\left(V_T s\, m_0 h_\vartheta' - \right.$$

$$\left.\left. - 2 V_T s\left(\left(\frac{m_0 h_z r_0^2 h^\vartheta}{\omega_c}\left(V_E^\vartheta + V_\parallel^\vartheta\right)\right)' - \cancel{\frac{m_0 h_z r_0^2 (h^\vartheta)^2}{\omega_c} V_\parallel'}\right)\right)\right] +$$

$$\left. + \bar{u}^3\left[\frac{V_T'}{V_T}s^2(-)\frac{2 m_0 h_z (h^\vartheta)^2 r_0^2 V_T s}{\omega_c} + \frac{s}{V_T}\left(- V_T^2 s^2\left(\frac{m_0 h_z r_0^2 (h^\vartheta)^2}{\omega_c}\right)'\right)\right]\right\} =$$

$$\frac{\partial f_0}{\partial p_z} = \gamma^{-1}\left(-\frac{f_0}{T}\right) m_0 V_T^2 \left\{ -\frac{1}{2}\left(\frac{\omega_c}{m_0 V_T^2}\right)'\left(\frac{m_0 V_T^2}{\omega_c}\right)\left[m_0 h_\vartheta - \frac{2 m_0 h_z h^\vartheta r_0^2}{\omega_c}\left(V_E^\vartheta + V_\parallel^\vartheta\right) - \right.\right.$$

$$\left. - \frac{2 m_0 h_z (h^\vartheta)^2 r_0^2 V_T S}{\omega_c}\bar{u}\right]\lambda^2 + \bar{u}^0\left[\left(\frac{n'}{n} - 3\frac{V_T'}{V_T}\right)\left(m_0 h_\vartheta - \frac{2 m_0 h_z h^\vartheta r_0^2}{\omega_c}\left(V_E^\vartheta + V_\parallel^\vartheta\right)\right)\right]$$

$$+ \bar{u}^1\left[-\left(\frac{n'}{n} - 3\frac{V_T'}{V_T}\right)\frac{2 m_0 h_z (h^\vartheta)^2 r_0^2 V_T S}{\omega_c} + \frac{S}{V_T} m_0 r_0 \omega_c h_z + \frac{S}{V_T} m_0 \left(V_{E\vartheta} + V_{\parallel\vartheta}\right)' \right.$$

$$\left. - \frac{S}{V_T}\left(\frac{m_0 h_z r_0^2}{\omega_c}\left(V_E^\vartheta + V_\parallel^\vartheta\right)^2\right)'\right] + \bar{u}^2\left[\frac{m_0 S^2}{V_T}\left(h_\vartheta V_T\right)' - \right.$$

$$\left.\left. - \frac{S^2}{V_T} 2\left(\frac{m_0 h_z h^\vartheta r_0^2 V_T}{\omega_c}\left(V_E^\vartheta + V_\parallel^\vartheta\right)\right)'\right] + \bar{u}^3\left[-\frac{S^3}{V_T}\left(\frac{m_0 h_z r_0^2 (h^\vartheta)^2 V_T^2}{\omega_c}\right)'\right]\right\}.$$

$$\vec{m}\cdot\frac{\partial f_0}{\partial \vec{J}} = \ell\frac{\partial f_0}{\partial J_\perp} + K_\vartheta\frac{\partial f_0}{\partial p_\vartheta} + K_z\frac{\partial f_0}{\partial p_z} = \gamma^{-1}\left(-\frac{f_0}{T}\right) m_0 V_T^2 \times$$

$$\left\{\ell\frac{m_0^2 r_0 \omega_c}{m_0 V_T^2}\omega_c + \frac{1}{2}\left(\frac{\omega_c}{m_0 V_T^2}\right)'\left(\frac{m_0 V_T^2}{\omega_c}\right)\left[m_0 r_0 K_\perp + \frac{2 m_0 h^\vartheta r_0^2 K_\parallel}{\omega_c} V^\vartheta + \right.\right.$$

$$\left. + \frac{2 m_0 (h^\vartheta)^2 r_0^2 V_T S K_\parallel}{\omega_c}\bar{u}\right]\lambda^2 + \bar{u}^0\left[\left(-\frac{n'}{n} + 3\frac{V_T'}{V_T}\right)\left(m_0 r_0 K_\perp + \frac{2 m_0 h^\vartheta r_0^2 K_\parallel}{\omega_c} V^\vartheta\right)\right]$$

$$+ \bar{u}^1\left[\left(-\frac{n'}{n} + 3\frac{V_T'}{V_T}\right)\frac{2 m_0 r_0^2 V_T (h^\vartheta)^2 S K_\parallel}{\omega_c} + \frac{S}{V_T} m_0 \omega_c r_0 K_\parallel - \frac{S}{V_T} m_0 \left(r_0 (\vec{K}\times\vec{V})_n\right)' - \right.$$

$$\left. - \frac{S}{V_T}\left(\frac{m_0 r_0^2 K_\parallel}{\omega_c}(V^\vartheta)^2\right)'\right] + \bar{u}^2\left[-\frac{m_0 S^2}{V_T}\left(r_0 K_\perp V_T\right)' - \right.$$

$$\left.\left. - \frac{2 S^2}{V_T} m_0 \left(\frac{h^\vartheta r_0^2 V_T K_\parallel}{\omega_c} V^\vartheta\right)'\right] + \bar{u}^3\left[-\frac{m_0 S^3}{V_T}\left(\frac{(h^\vartheta)^2 r_0^2 V_T^2 K_\parallel}{\omega_c}\right)'\right]\right\}.$$

$$\vec{V} = \vec{V}_E + \vec{V}_\parallel, \quad \vec{K} = K_\vartheta \vec{e}_\vartheta + K_z \vec{e}_z = K_\perp \vec{e}_\perp + K_\parallel \vec{e}_\parallel$$

$$\vec{m}\cdot\frac{\partial f_0}{\partial \vec{J}} = \gamma^{-1}\left(-\frac{f_0}{T}\right) m_0 V_T^2 \left\{\frac{m_0 r_0 \omega_c}{V_T^2}\ell\omega_c + P_{0a} + P_{02}\lambda^2 + P_{12}\bar{u}'\lambda^2 + \right.$$

$$\left. + P_{10}\bar{u}' + P_{20}(\bar{u}')^2 + P_{30}(\bar{u}')^3\right\}$$

$$\Omega^{\phi} = \omega_c$$

$$\Omega^{\vartheta} = h^{\vartheta}(\bar{u} v_T s + V_{\parallel}) + V_E^{\vartheta} - \frac{h_z}{\omega_c}\left(h^{\vartheta}\bar{u} v_T s + h^{\vartheta} V_{\parallel} + V_E^{\vartheta}\right)^2 +$$

$$+ \frac{h_z \omega_c'}{\cancel{m_0}\omega_c r_0}\,\frac{1}{2}\,\frac{\cancel{m_0} v_T^2}{\omega_c}\lambda^2 = V^{\vartheta} + h^{\vartheta} v_T s \bar{u} - \frac{h_z}{\omega_c}\Big[(h^{\vartheta})^2 \bar{u}^2 v_T^2 s^2 +$$

$$+ 2 h^{\vartheta}\bar{u} v_T s V^{\vartheta} + (V^{\vartheta})^2\Big] + \frac{h_z \omega_c' v_T^2}{2 r_0 \omega_c^2}\lambda^2 =$$

$$= V^{\vartheta} - \frac{h_z}{\omega_c}(V^{\vartheta})^2 + \bar{u}\left[h^{\vartheta} v_T s - \frac{2 h_z h^{\vartheta} v_T s V^{\vartheta}}{\omega_c}\right] - \frac{h_z (h^{\vartheta})^2 v_T^2 s^2}{\omega_c}\bar{u}^2$$

$$+ \frac{1}{2}\,\frac{h_z \omega_c' v_T^2}{r_0 \omega_c^2}\lambda^2.$$

$$\Omega^{z} = h^{z}(\bar{u} v_T s + V_{\parallel}) + V_E^{z} + \frac{h_{\vartheta}}{\omega_c}\left(h^{\vartheta}\bar{u} v_T s + h^{\vartheta} V_{\parallel} + V_E^{\vartheta}\right)^2 -$$

$$- \frac{h^{\vartheta}\omega_c' r_0}{\cancel{m_0}\omega_c}\,\frac{1}{2}\,\frac{\cancel{m_0} v_T^2}{\omega_c}\lambda^2 = V^{z} + h^{z} v_T s \bar{u} + \frac{h_{\vartheta}}{\omega_c}\Big((h^{\vartheta})^2 \bar{u}^2 v_T^2 s^2 +$$

$$+ 2 h^{\vartheta}\bar{u} v_T s V^{\vartheta} + (V^{\vartheta})^2\Big) - \frac{1}{2}\,\frac{h^{\vartheta}\omega_c' r_0 v_T^2}{\omega_c^2}\lambda^2 =$$

$$= V^{z} + \frac{h^{\vartheta} r_0^2}{\omega_c}(V^{\vartheta})^2 + \bar{u}\left[h^{z} v_T s + \frac{2(\hat{h}^{\vartheta})^2 V^{\vartheta} v_T s}{\omega_c}\right] + \frac{(\hat{h}^{\vartheta})^3 v_T^2 s^2}{r_0 \omega_c}\bar{u}^2 -$$

$$- \frac{1}{2}\,\frac{h^{\vartheta}\omega_c' r_0 v_T^2}{\omega_c^2}\lambda^2.$$

$$\vec{m}\cdot\vec{\Omega} - \omega = l\omega_c + k_{\vartheta}\Omega^{\vartheta} + k_z \Omega^{z} - \omega = l\omega_c - \omega + \vec{k}\cdot\vec{V} - \frac{r_0 k_{\perp}}{\omega_c}(V^{\vartheta})^2 +$$

$$+ \bar{u}\left[k_{\parallel} v_T s - \frac{2 h^{\vartheta} v_T s k_{\perp} r_0 V^{\vartheta}}{\omega_c}\right] - \frac{(h^{\vartheta})^2 v_T^2 s^2 r_0 k_{\perp}}{\omega_c}\bar{u}^2 +$$

$$+ \frac{1}{2}\,\frac{\omega_c' v_T^2 k_{\perp}}{\omega_c^2}\lambda^2$$

$$\Omega^{\beta} = Q^{\beta}_{00} + Q^{\beta}_{10}\bar{u}' + Q^{\beta}_{20}(\bar{u}')^2 + Q^{\beta}_{02}\lambda^2$$

$$\vec{m}\cdot\vec{\Omega}-\omega = l\omega_c-\omega+\vec{k}\cdot\vec{V}-\frac{r_0 k_\perp}{\omega_c}(V^\vartheta)^2+\frac{1}{2}\frac{k_\perp V_T^2\omega_c'}{\omega_c^2}\lambda^2+$$
$$+\bar{u}\left[k_\parallel V_T-\frac{2h^\vartheta V^\vartheta V_T k_\perp r_0}{\omega_c}\right]\cdot s-\bar{u}^2\left[\frac{(h^\vartheta)^2 V_T^2 r_0 k_\perp}{\omega_c}\right]\cdot s^2=$$

$$=-\nu x_2+\nu x_1 s\bar{u}-\nu x_3 s^2\bar{u}^2=\nu\left[-x_2+x_1 s\bar{u}-x_3 s^2\bar{u}^2\right]=$$

$$=-\nu\left[(\bar{x}_2-i\gamma)s^{-2}-\bar{x}_1 s^{-2}\bar{u}+\bar{x}_3 s^{-2}\bar{u}^2\right]$$

$$\bar{x}_1=x_1 s^3,\quad \bar{x}_2=\frac{x_2+i\alpha}{1+2\alpha}=x_2 s^2+i\gamma,\quad \bar{x}_3=x_3 s^{+4}.$$

$$R_{00}=R_{00}'+l\omega_c$$
$$P_{00}=\frac{m_0 r_0\omega_c}{V_T^2}l\omega_c+P_{00}'$$

$$\left[\left(\vec{m}\cdot\frac{\partial f_0}{\partial\vec{J}}\right)\Omega^\beta-\left(\vec{m}\cdot\vec{\Omega}-\omega\right)\frac{\partial f_0}{\partial J_\perp}\right]_{\bar{u}'}=\ ?$$

$$\vec{m}\cdot\vec{\Omega}-\omega=R_{00}'+R_{10}(\bar{u}')+R_{20}(\bar{u}')^2+R_{02}\lambda^2+l\omega_c$$

$$\left(\vec{m}\cdot\frac{\partial f_0}{\partial\vec{J}}\right)=y^{-1}\left(-\frac{f_0}{T}\right)_{\bar{u}'}m_0V_T^2\left\{\frac{m_0r_0\omega_c}{V_T^2}l\omega_c+P_{00}'+P_{02}\lambda^2+P_{12}(\bar{u}')\lambda^2+\right.$$
$$\left.+P_{10}(\bar{u}')+P_{20}(\bar{u}')^2+P_{30}(\bar{u}')^3\right\}.$$

$$\Omega^\beta=Q_{00}^\beta+Q_{10}^\beta(\bar{u}')+Q_{20}^\beta(\bar{u}')^2+Q_{02}^\beta\lambda^2.$$

$$\frac{\partial f_0}{\partial J_\beta}=y^{-1}\left(-\frac{f_0}{T}\right)_{\bar{u}'}m_0V_T^2\left\{Z_{00}^\beta+Z_{02}^\beta\lambda^2+Z_{12}^\beta(\bar{u}')\lambda^2+Z_{10}^\beta(\bar{u}')+Z_{20}^\beta(\bar{u}')^2+\right.$$
$$\left.+Z_{30}^\beta(\bar{u}')^3\right\}$$

$$[\ ]_{\bar{u}'}^\beta=y^{-1}\left(-\frac{f_0}{T}\right)_{\bar{u}'}m_0V_T^2\left\{\left[\frac{m_0r_0\omega_c}{V_T^2}l\omega_c+P_{00}'+P_{02}\lambda^2+P_{12}(\bar{u}')\lambda^2+\right.\right.$$
$$\left.+P_{10}(\bar{u}')+P_{20}(\bar{u}')^2+P_{30}(\bar{u}')^3\right]\left[Q_{00}^\beta+Q_{10}^\beta(\bar{u}')+Q_{20}^\beta(\bar{u}')^2+Q_{02}^\beta\lambda^2\right]-$$
$$-\left[R_{00}'+R_{10}(\bar{u}')+R_{20}(\bar{u}')^2+R_{02}\lambda^2+l\omega_c\right]\left[Z_{00}^\beta+Z_{02}^\beta\lambda^2+Z_{12}^\beta(\bar{u}')\lambda^2+\right.$$
$$\left.\left.+Z_{10}^\beta(\bar{u}')+Z_{20}^\beta(\bar{u}')^2+Z_{30}^\beta(\bar{u}')^3\right]\right\}.$$

$$F^{\beta} = [\;\;]^{\beta}_{\bar{u}'} = \gamma^{-1}\left(-\frac{f_0}{T}\right)_{\bar{u}'} m_0 V_T^2 \Big\{ P_{00} Q^{\beta}_{00} - R_{00} Z^{\beta}_{00} +$$

$$+ \lambda^2 \left[P_{02} Q^{\beta}_{00} + P_{00} Q^{\beta}_{02} - R_{02} Z^{\beta}_{00} - R_{00} Z^{\beta}_{02} \right] + \lambda^4 \left[P_{02} Q^{\beta}_{02} - R_{02} Z^{\beta}_{02} \right]$$

$$+ \bar{u}' \left[P_{10} Q^{\beta}_{00} + P_{00} Q^{\beta}_{10} - R_{10} Z^{\beta}_{00} - R_{00} Z^{\beta}_{10} \right] + \bar{u}'\lambda^2 \Big[P_{12} Q^{\beta}_{00} +$$

$$+ P_{10} Q^{\beta}_{02} + P_{02} Q^{\beta}_{10} - R_{10} Z^{\beta}_{02} - R_{02} Z^{\beta}_{10} - R_{00} Z^{\beta}_{12} \Big] + \bar{u}'\lambda^4 \Big[P_{12} Q^{\beta}_{02} -$$

$$- R_{02} Z^{\beta}_{12} \Big] + \bar{u}'^2 \left[P_{20} Q^{\beta}_{00} + P_{10} Q^{\beta}_{10} + P_{00} Q^{\beta}_{20} - R_{20} Z^{\beta}_{00} - R_{10} Z^{\beta}_{10} - R_{00} Z^{\beta}_{20} \right] +$$

$$+ \bar{u}'^2 \lambda^2 \left[P_{20} Q^{\beta}_{02} + P_{12} Q^{\beta}_{10} + P_{02} Q^{\beta}_{20} - R_{20} Z^{\beta}_{02} - R_{10} Z^{\beta}_{12} - R_{02} Z^{\beta}_{20} \right] +$$

$$+ \bar{u}'^4 \left[P_{30} Q^{\beta}_{10} + P_{20} Q^{\beta}_{20} - R_{20} Z^{\beta}_{20} - R_{10} Z^{\beta}_{30} \right] + \bar{u}'^5 \left[P_{30} Q^{\beta}_{20} - R_{20} Z^{\beta}_{30} \right]$$

$$+ \bar{u}'^3 \left[P_{30} Q^{\beta}_{00} + P_{20} Q^{\beta}_{10} + P_{10} Q^{\beta}_{20} - R_{20} Z^{\beta}_{10} - R_{10} Z^{\beta}_{20} - R_{00} Z^{\beta}_{30} \right] +$$

$$+ \bar{u}'^3 \lambda^2 \left[P_{30} Q^{\beta}_{02} + P_{12} Q^{\beta}_{20} - R_{20} Z^{\beta}_{12} - R_{02} Z^{\beta}_{30} \right] \Big\} + \bar{u}'^5 \left(P_{30} Q^{\beta}_{20} - R_{20} Z^{\beta}_{30} \right)$$

$$F^{\beta} = \gamma^{-1}\left(-\frac{f_0}{T}\right)_{\bar{u}'} m_0 V_T^2 \Big\{ F^{\beta}_{00} + F^{\beta}_{02}\lambda^2 + F^{\beta}_{04}\lambda^4 + F^{\beta}_{10}(\bar{u}') +$$

$$+ F^{\beta}_{12}(\bar{u}')\lambda^2 + F^{\beta}_{14}(\bar{u}')\lambda^4 + F^{\beta}_{20}(\bar{u}')^2 + F^{\beta}_{22}(\bar{u}')^2\lambda^2 +$$

$$+ F^{\beta}_{30}(\bar{u}')^3 + F^{\beta}_{32}(\bar{u}')^3\lambda^2 + F^{\beta}_{40}(\bar{u}')^4 + F^{\beta}_{50}(\bar{u}')^5 \Big\}.$$

$$\sigma^{kj}_{nn'} = \sum_{\ell} \int_0^{\infty} d\lambda\, 2\lambda \frac{1}{2}\frac{m_0 v_T^2}{\omega_c} \int_{-\infty}^{+\infty} d\bar{u}\, v_T s\, \mathcal{Y} [a^k_\alpha(n)]^*_\ell\, \Omega^\alpha [a^j_\beta(n')]_\ell f^\beta_{\vec{m}}$$

$$[a^k_\alpha(n)]^*_\ell = \frac{(-i)^n}{n!} A^k_\alpha(n) (-i)^{s_1(k,\alpha)} (-i)^{s_2(k,\alpha)} \frac{\partial^{s_1(k,\alpha)}}{\partial q_1^{s_1(k,\alpha)}} \frac{\partial^{s_2(k,\alpha)}}{\partial q_2^{s_2(k,\alpha)}} \frac{\partial^n}{\partial q_2^n} J_\ell(\tilde{\varkappa}\rho) e^{-i\ell\tilde{\alpha}}$$

$$[a^j_\beta(n')]_\ell = \frac{i^{n'}}{n'!} A^j_\beta(n')\, i^{s_1(j,\beta)} \cdot i^{s_2(j,\beta)} \cdot \frac{\partial^{s_1(j,\beta)}}{\partial p_1^{s_1(j,\beta)}} \frac{\partial^{s_2(j,\beta)}}{\partial p_2^{s_2(j,\beta)}} \frac{\partial^{n'}}{\partial p_2^{n'}} J_\ell(\varkappa\rho) e^{i\ell\alpha}$$

see page 28'' of old notes about KiLCA.

$$\cancel{\sigma^{kj}_{nn'}(\ell) = \int_0^{\infty} d\lambda\, 2\lambda \frac{1}{2}\frac{m_0 v_T^2}{\omega_c} \int_{-\infty}^{+} d\bar{u}\, v_T s\, \mathcal{Y}}$$

$$\sigma^{kj}_{nn'}(\ell) = (-)^{n+s_1(k,\alpha)+s_2(k,\alpha)}\, i^{n+n'+s_1(k,\alpha)+s_2(k,\alpha)+s_1(j,\beta)+s_2(j,\beta)} \times$$

$$\times \frac{1}{n!}\frac{1}{n'!} A^k_\alpha(n) A^j_\beta(n') \frac{\partial^{s_1(k,\alpha)}}{\partial q_1^{s_1(k,\alpha)}} \frac{\partial^{n+s_2(k,\alpha)}}{\partial q_2^{n+s_2(k,\alpha)}} \frac{\partial^{s_1(j,\beta)}}{\partial p_1^{s_1(j,\beta)}} \frac{\partial^{n'+s_2(j,\beta)}}{\partial p_2^{n'+s_2(j,\beta)}}$$

$$e^{i\ell(\alpha-\tilde{\alpha})} \int_0^{\infty} d\lambda\, \cancel{2}\lambda \cancel{\tfrac{1}{2}} \frac{m_0 v_T^2}{\omega_c} \int_{-\infty}^{+\infty} d\bar{u}\, v_T s\, \mathcal{Y}\; J_\ell\left(\tilde{\varkappa}\frac{v_T}{\omega_c}\lambda\right) J_\ell\left(\varkappa\frac{v_T}{\omega_c}\lambda\right)$$

$$\left[Q^\alpha_{00} + Q^\alpha_{10}\bar{u} + Q^\alpha_{20}\bar{u}^2 + Q^\alpha_{02}\lambda^2\right] \times i\left(-\frac{1}{\nu}\right)(1+2\alpha)^{-1} e^{\frac{\gamma}{2}\bar{u}^2} \times$$

$$\times \int_{-\infty}^{+\infty} d\bar{u}'\, e^{-\frac{\gamma}{2}\bar{u}'^2}\, \mathcal{Y}^{-1} (-)\frac{1}{(2\pi)^{3/2}} \frac{n_0}{(m_0^3 v_T^3)} e^{-\frac{1}{2}\lambda^2} e^{-\frac{1}{2}\bar{u}'^2 s^2}$$

$$\left\{\sum_{\mu,\nu} F^\beta_{\mu\nu}(\ell) (\bar{u}')^\mu \lambda^\nu\right\} (-)\frac{1}{\sqrt{2\pi}} \int_0^{\infty} d\tau \left(1-e^{-2\tau}\right)^{-\frac{1}{2}} e^{i\bar{x}_1(\bar{u}-\bar{u}') + (i\bar{x}_2 - \bar{x}_1^2)\tau} \times$$

$$e^{-\frac{1}{2}(1-e^{-2\tau})^{-1}\left[\bar{u}-\bar{u}'e^{-\tau}+2i\bar{x}_1(1-e^{-\tau})\right]^2} \qquad G(\bar{u},\bar{u}')$$

$$\sigma_{hh'}^{kj}(\ell) = (-)^{h+s_1(k,\alpha)+s_2(k,\alpha)}\, i^{h+h'+s_1(k,\alpha)+s_2(k,\alpha)+s_1(j,\beta)+s_2(j,\beta)} \times$$ (32)

$$\times \frac{1}{h!}\frac{1}{h'!} A^k_\alpha(h) A^j_\beta(h') \frac{\partial^{s_1(k,\alpha)}}{\partial q_1^{s_1(k,\alpha)}} \frac{\partial^{h+s_2(k,\alpha)}}{\partial q_2^{h+s_2(k,\alpha)}} \frac{\partial^{s_1(j,\beta)}}{\partial p_1^{s_1(j,\beta)}} \frac{\partial^{h'+s_2(j,\beta)}}{\partial p_2^{h'+s_2(j,\beta)}} e^{i\ell(\alpha-\tilde\alpha)} \times$$

$$\times \frac{m_0 V_T^2}{\omega_c} V_T s\, i\left(-\frac{1}{\nu}\right)(1+2\alpha)^{-1}(-)\frac{1}{(2\pi)^{3/2}}\frac{n_0}{m_0^3 V_T^3} \times \int_0^\infty d\lambda\,\lambda \int_{-\infty}^{+\infty} d\bar u\, J_\ell\left(\tilde\varkappa \frac{V_T}{\omega_c}\lambda\right)\cdot$$

$$\cdot J_\ell\left(\varkappa\frac{V_T}{\omega_c}\lambda\right)\left[Q^\alpha_{00}+Q^\alpha_{10}\bar u + Q^\alpha_{20}\bar u^2 + Q^\alpha_{02}\lambda^2\right] e^{\frac{\gamma}{2}\bar u^2}\int_{-\infty}^{+\infty} d\bar u'\, e^{-\frac{\gamma}{2}\bar u'^2} \times$$

$$\times e^{-\frac{1}{2}\bar u'^2 s^2}\, e^{-\frac{1}{2}\lambda^2}\sum_{\mu,\nu} F^\beta_{\mu\nu}(\ell)(\bar u')^\mu \lambda^\nu\, (-)\frac{1}{\sqrt{2\pi}}\int_0^\infty d\tau\left(1-e^{-2\tau}\right)^{-\frac{1}{2}} \times$$

$$\times e^{i\tau\bar x_2}\exp\left\{i\bar x_1(\bar u-\bar u') - \bar x_1^2\tau - \frac{1}{2}(1-e^{-2\tau})^{-1}\left[\bar u - \bar u' e^{-\tau} + 2i\bar x_1(1-e^{-\tau})\right]^2\right\}.$$

$$= \ldots \frac{1}{(2\pi)^{3/2}}\frac{n_0}{m_0^2\omega_c}\, i\frac{s}{\nu}(1+2\alpha)^{-1} \times \int_0^\infty d\lambda\,\lambda\, J_\ell\left(\tilde\varkappa\frac{V_T}{\omega_c}\lambda\right) J_\ell\left(\varkappa\frac{V_T}{\omega_c}\lambda\right) \times$$

$$\times e^{-\frac{1}{2}\lambda^2}\int_{-\infty}^{+\infty} d\bar u \int_{-\infty}^{+\infty} d\bar u' \left[Q^\alpha_{00}+Q^\alpha_{10}\bar u + Q^\alpha_{20}\bar u^2 + Q^\alpha_{02}\lambda^2\right] \times$$

$$\times \sum_{\mu,\nu} F^\beta_{\mu\nu}(\ell)(\bar u')^\mu \lambda^\nu\, e^{\frac{\gamma}{2}\bar u^2 - \frac{\gamma}{2}\bar u'^2 - \frac{1}{2}\bar u'^2 s^2} \times$$

$$\times (-)\frac{1}{\sqrt{2\pi}}\int_0^\infty d\tau\left(1-e^{-2\tau}\right)^{-\frac{1}{2}} e^{i\tau\left(i\gamma + s^2 x_2' - \frac{1}{2}\bar x_4 \lambda^2\right)}.$$

$$\cdot \exp\left\{i\bar x_1(\bar u - \bar u') - \bar x_1^2\tau - \frac{1}{2}(1-e^{-2\tau})^{-1}\left[\bar u - \bar u' e^{-\tau} + 2i\bar x_1(1-e^{-\tau})\right]^2\right\}.$$

$$\bar x_2 = x_2 s^2 + i\gamma = x_2' s^2 + i\gamma - \frac{1}{2}\frac{k_\perp V_T^2\omega_c'}{\nu\,\omega_c^2}\lambda^2 s^2 = x_2' s^2 + i\gamma - \frac{1}{2}\bar x_4\lambda^2.$$

$$x_2' = x_2 \big|(\omega_c' = 0)$$

$$\frac{\alpha(\bar u^2 - \bar u'^2) - \bar u'^2}{1+2\alpha} = \frac{\alpha\bar u^2 - \alpha\bar u'^2 - \bar u'^2 - \alpha\bar u'^2 + \alpha\bar u'^2}{1+2\alpha} = \gamma\bar u'^2 - \bar u'^2 + \gamma\bar u^2$$

$$= \gamma(\bar u^2 + \bar u'^2) - \bar u'^2.$$

$$\mathcal{Y}\mathcal{J}=\int_0^\infty d\lambda\, \lambda^{2\delta+1}\, \mathcal{J}_\ell(\tilde{\varkappa}\rho_L\lambda)\, \mathcal{J}_\ell(\varkappa\rho_L\lambda)\, e^{-\frac{1}{2}\lambda^2}\, e^{-\frac{1}{2}i\tau\bar{x}_4\lambda^2} =$$

$$=(-)^\delta \frac{\partial^\delta}{\partial x^\delta}\left[\frac{1}{2x}\, e^{-\frac{\tilde{\varkappa}^2+\varkappa^2}{4x}\rho_L^2}\, I_\ell\left(\frac{\tilde{\varkappa}\varkappa}{2x}\rho_L^2\right)\right]_{x=\frac{1}{2}(1+i\tau\bar{x}_4)}$$

Simplified approach is to expand $\mathcal{J}_\ell$ in series:

$$\begin{cases} \mathcal{J}_\ell(\varkappa\rho_L\lambda)=\sum\limits_{b=0}^{\infty} \mathcal{Y}_\ell^b\, \lambda^{\ell+2b}\,, \quad \mathcal{J}_\ell(\tilde{\varkappa}\rho_L\lambda)=\sum\limits_{\tilde{b}=0} \tilde{\mathcal{J}}_\ell^{\tilde{b}}\, \lambda^{\ell+2\tilde{b}} \\ \mathcal{Y}_\ell^b = \dfrac{(-)^b}{b!\,(\ell+b)!}\left(\dfrac{1}{2}\right)^{2b+\ell}\left(\varkappa\rho_L\right)^{2b+\ell}, \\ \tilde{\mathcal{J}}_\ell^{\tilde{b}} = \dfrac{(-)^{\tilde{b}}}{\tilde{b}!\,(\ell+\tilde{b})!}\left(\dfrac{1}{2}\right)^{\ell+2\tilde{b}}\left(\tilde{\varkappa}\rho_L\right)^{\ell+2\tilde{b}}. \end{cases}$$

$$\mathcal{Y}\mathcal{J}=\int_0^\infty d\lambda\, \lambda^{2\delta+1}\sum_{b=0}^{N_b}\mathcal{Y}_\ell^b\,\lambda^{\ell+2b}\sum_{\tilde{b}=0}^{N_b}\tilde{\mathcal{J}}_\ell^{\tilde{b}}\,\lambda^{\ell+2\tilde{b}}\; e^{-\frac{1}{2}(1+i\tau\bar{x}_4)\lambda^2}=$$

$$=\sum_{b=0}^{N_b}\sum_{\tilde{b}=0}^{N_b}\mathcal{Y}_\ell^b\,\tilde{\mathcal{J}}_\ell^{\tilde{b}}\int_0^\infty d\lambda\,\lambda^{2\delta+1+2\ell+2(b+\tilde{b})}\; e^{-\frac{1}{2}(1+i\tau\bar{x}_4)\lambda^2}=$$

$$=\sum_{b=0}^{N_b}\sum_{\tilde{b}=0}^{N_b}\mathcal{Y}_\ell^b\,\tilde{\mathcal{J}}_\ell^{\tilde{b}}\; 2^{\delta+\ell+b+\tilde{b}}\left(1+i\tau\bar{x}_4\right)^{-(\delta+\ell+b+\tilde{b}+1)}=$$

$$=\sum_{b=0}^{N_b}\sum_{\tilde{b}=0}^{N_b}\mathcal{Y}_{\ell b}^{(0)}\,\tilde{\mathcal{J}}_{\ell\tilde{b}}^{(0)}\; 2^{\delta+\ell+b+\tilde{b}}\left(1+i\tau\bar{x}_4\right)^{-(\delta+\ell+b+\tilde{b}+1)}$$

$$\mathcal{Y}_{\ell b}^{(p)}=\frac{\partial^p}{\partial\ldots}\,\mathcal{Y}_\ell^b \quad \text{– later we'll need derivatives}$$

$$\sigma^{kj}_{nn'}(\ell) = \ldots \frac{1}{(2\pi)^{3/2}} \frac{n_0}{m_0^2 \omega_c} \, i \frac{s}{\nu} (1+2\alpha)^{-1} (-) \frac{1}{\sqrt{2\pi}} \times$$

$$\times \int_0^\infty d\tau \int_{-\infty}^{+\infty} d\bar{u} \int_{-\infty}^{+\infty} d\bar{u}' \sum_{\mu,\nu} F^{\beta}_{\mu 2\nu}(\ell) \Big\{ \Big[Q^{\alpha}_{00} + Q^{\alpha}_{10}\bar{u} + Q^{\alpha}_{20}\bar{u}^2 \Big] (\bar{u}')^{\mu} \times$$

$$\times \sum_{b=0}^{N_b} \sum_{\tilde{b}=0}^{N_b} \mathcal{J}^{(0)}_{\ell b} \tilde{\mathcal{J}}^{(0)}_{\ell \tilde{b}} \, 2^{\nu+\ell+b+\tilde{b}} (1+i\tau\bar{x}_4)^{-(\nu+\ell+b+\tilde{b}+1)} +$$

$$+ Q^{\alpha}_{02} (\bar{u}')^{\mu} \times \sum_{b=0}^{N_b} \sum_{\tilde{b}=0}^{N_b} \mathcal{J}^{(0)}_{\ell b} \tilde{\mathcal{J}}^{(0)}_{\ell \tilde{b}} \, 2^{\nu+1+\ell+b+\tilde{b}} (1+i\tau\bar{x}_4)^{-(\nu+1+\ell+b+\tilde{b}+1)} \Big\} \times$$

$$\cancel{\times e^{\gamma(\bar{u}^2+\bar{u}'^2)/2 - \bar{u}'^2/2}} \times e^{\frac{\gamma}{2}(\bar{u}^2+\bar{u}'^2) - \frac{1}{2}\bar{u}'^2} \times (1-e^{-2\tau})^{-1/2} \times$$

$$\times e^{i\tau(i\gamma + s^2 x_2')} \times e^{i\bar{x}_1(\bar{u}-\bar{u}') - \bar{x}_1^2\tau - \frac{1}{2}(1-e^{-2\tau})^{-1}[\bar{u}-\bar{u}'e^{-\tau}+2i\bar{x}_1(1-e^{-\tau})]^2} =$$

$$\cancel{= \ldots \frac{1}{(2\pi)^{3/2}} \frac{n_0}{m_0^2 \omega_c} i \frac{s}{\nu}}$$

We introduce special function as follows:

$$I^{mn}_{\ell} = (1+2\alpha)^{-\frac{3}{2}} (1+2\alpha)^{-\frac{m+n}{2}} (-) \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} d\bar{u} \int_{-\infty}^{+\infty} d\bar{u}' \int_0^{+\infty} d\tau$$

$$(1+i\bar{x}_4\tau)^{-(\ell+1)} (1-e^{-2\tau})^{-\frac{1}{2}} e^{\frac{\gamma}{2}(\bar{u}^2+\bar{u}'^2) - \frac{1}{2}\bar{u}'^2} \times$$

$$\times \exp\Big\{ i\bar{x}_1(\bar{u}-\bar{u}') - \bar{x}_1^2\tau - \frac{1}{2}(1-e^{-2\tau})^{-1} \big[\bar{u}-\bar{u}'e^{-\tau}+2i\bar{x}_1(1-e^{-\tau})\big]^2 \Big\} \times$$

$$\times (\bar{u})^m (\bar{u}')^n \times \underbrace{e^{\tau(-\gamma + is^2 x_2')}}_{e^{i\tau \bar{x}_2'}}$$

$$e^{\tau(i\bar{x}_2' - \bar{x}_1^2)}$$

$$I_{\ell}^{mn} = (1+2\alpha)^{-(m+n+3)/2} \frac{(-)}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} d\bar{u} \int_{-\infty}^{+\infty} d\bar{u}' \int_{0}^{\infty} d\tau \; \times$$

$$\times \left(1+i\bar{x}_4\tau\right)^{-(\ell+1)} \left(1-e^{-2\tau}\right)^{-\frac{1}{2}} \frac{\partial^m}{\partial x^m} \frac{\partial^n}{\partial y^n} \exp\Big[x\bar{u} + y\bar{u}' - \tfrac{1}{2}\bar{u}'^2 +$$

$$+ \frac{\gamma}{2}\left(\bar{u}^2 + \bar{u}'^2\right) + \tau\left(i\bar{x}_2' - \bar{x}_1^2\right) + i\bar{x}_1\left(\bar{u}-\bar{u}'\right) - \frac{1}{2}\left(1-e^{-2\tau}\right)^{-1}\left[\bar{u} - \bar{u}'e^{-\tau} +\right.$$

$$\left.+ 2i\bar{x}_1\left(1-e^{-\tau}\right)\right]^2 \Big]$$

$$\sigma^{kj}_{nn'}(\ell) = \,,,,\, \frac{1}{(2\pi)^{3/2}} \frac{n_0}{m_0^2 \omega_c} \frac{i}{\nu} \sum_{\mu,\nu}{}' F^{\beta}_{\mu 2 \nu}(\ell) \times$$

$$\Big\{ \sum_{b,\tilde b}{}' \mathcal{J}_{\ell b} \tilde{\mathcal{J}}_{\ell \tilde b}\, 2^{\nu+\ell+b+\tilde b} \Big[Q^{\alpha}_{00} I^{0\mu}_{\nu+\ell+b+\tilde b} + Q^{\alpha}_{10} I^{1\mu}_{\nu+\ell+b+\tilde b} +$$

$$+ Q^{\alpha}_{20} I^{2\mu}_{\nu+\ell+b+\tilde b} \Big] + \sum_{b,\tilde b}{}' \mathcal{J}_{\ell b} \tilde{\mathcal{J}}_{\ell b}\, 2^{\nu+1+\ell+b+\tilde b} Q^{\alpha}_{02} I^{0\mu}_{\nu+1+\ell+b+\tilde b} \Big\} =$$

$$= \,,,,\, \frac{1}{(2\pi)^{3/2}} \frac{n_0}{m_0^2 \omega_c} \frac{i}{\nu} \sum_{\mu,\nu}{}' F^{\beta}_{\mu 2\nu}(\ell) \sum_{b,\tilde b}{}' \mathcal{J}_{\ell b} \tilde{\mathcal{J}}_{\ell \tilde b}\, 2^{\nu+\ell+b+\tilde b} \Big[Q^{\alpha}_{00} I^{0\mu}_{\nu+\ell+b+\tilde b} +$$

$$+ Q^{\alpha}_{10} I^{1\mu}_{\nu+\ell+b+\tilde b} + Q^{\alpha}_{20} I^{2\mu}_{\nu+\ell+b+\tilde b} + 2 Q^{\alpha}_{02} I^{0\mu}_{\nu+1+\ell+b+\tilde b} \Big].$$

Let we introduce the following functions:

$$D^{mn}_{\ell b}(k_\perp \rho_L) = i^{m+n} \rho_L^{-m-n} \frac{\partial^m}{\partial \varkappa_1^m} \frac{\partial^n}{\partial \varkappa_2^n} e^{i\ell\varphi} \mathcal{J}_{\ell b}(\varkappa \rho_L) \Bigg|_{\substack{\varkappa = \sqrt{\varkappa_1^2 + \varkappa_2^2} \\ \varphi = \operatorname{atan}\left(\frac{\varkappa_2}{\varkappa_1}\right) \\ \varkappa_1 = k_\perp \\ \varkappa_2 = 0}} =$$

$$= \rho_L^{-(m+n)} i^{m+n} \frac{(-)^b}{b!(\ell+b)!} \left(\frac{1}{2}\right)^{\ell+2b} \times$$

$$\times \frac{\partial^m}{\partial \varkappa_1^m} \frac{\partial^n}{\partial \varkappa_2^n} e^{i\ell\varphi} (\varkappa \rho_L)^{\ell+2b} \Bigg|_{\{\varkappa_1 = k_\perp,\ \varkappa_2 = 0\}}$$

Then:

$$\sigma^{kj}_{nn'}(\ell) = \frac{1}{(2\pi)^{3/2}} \frac{n_0}{m_0^2 \omega_c} \frac{i}{\nu} \frac{A^{k}_{\alpha}(n) A^{j}_{\beta}(n')}{n!\, n'!} \sum_{\mu,\nu} F^{\beta}_{\mu 2\nu}(\ell) \sum_{b,\tilde b}{}' 2^{\nu+\ell+b+\tilde b} \times$$

$$\times \Big[D^{s_1(k,\alpha)\, n+s_2(k,\alpha)}_{\ell \tilde b}(k_\perp \rho_L) \Big]^{*} D^{s_1(j,\beta)\, n'+s_2(j,\beta)}_{\ell b}(k_\perp \rho_L)\, \rho_L^{\,n+s_1(k,\alpha)+s_2(k,\alpha)+n'+s_1(j,\beta)+s_2(j,\beta)} \times$$

$$\Big[Q^{\alpha}_{00} I^{0\mu}_{\nu+\ell+b+\tilde b} + Q^{\alpha}_{10} I^{1\mu}_{\nu+\ell+b+\tilde b} + Q^{\alpha}_{20} I^{2\mu}_{\nu+\ell+b+\tilde b} + 2 Q^{\alpha}_{02} I^{0\mu}_{\nu+1+\ell+b+\tilde b} \Big].$$

Energy conservation

KE without energy conservation has a solution:

$$f_{\vec{m}}^{OU}(v_\perp, v_\parallel) = \hat{L}^{-1} Q(v_\perp, v_\parallel) = f_{\vec{m}}^{OU}(\lambda, \bar{u}) = \int_{-\infty}^{+\infty} d\bar{u}' \, \hat{G}(\lambda, \bar{u}, \bar{u}') Q_{\vec{m}}(\lambda, \bar{u}')$$

$$\hat{G}(\lambda, \bar{u}, \bar{u}') = -\frac{1}{\nu}(1+2\alpha)^{-1} e^{\frac{\gamma}{2}(\bar{u}^2 - \bar{u}'^2)} \, G(\lambda, \bar{u}, \bar{u}')$$

$$\bar{u} = \frac{u_\parallel - V_\parallel}{V_T}\sqrt{1+2\alpha}, \quad \gamma = \frac{\alpha}{1+2\alpha}, \quad \lambda = \frac{v_\perp}{v_T}.$$

$$G(\lambda, \bar{u}, \bar{u}') = -\frac{1}{\sqrt{2\pi}} \int_0^\infty d\tau \left(1 - e^{-2\tau}\right)^{-\frac{1}{2}} e^{i\bar{x}_1(\bar{u} - \bar{u}') + (i\bar{x}_2' - \bar{x}_1^2)\tau} \times$$

$$\times e^{-\frac{1}{2} i \bar{x}_4 \lambda^2 \tau} \times e^{-\frac{1}{2}(1 - e^{-2\tau})^{-1}\left[\bar{u} - \bar{u}' e^{-\tau} + 2i\bar{x}_1(1 - e^{-\tau})\right]^2}$$

$$\bar{x}_2' = \frac{x_2'}{1+2\alpha} + i\gamma, \quad \bar{x}_4 = \frac{x_4}{1+2\alpha}, \quad x_4 = \frac{k_\perp \omega_c' v_T^2}{\nu \omega_c^2}.$$

We start from eq. (60):

$$f_{\vec{m}}(v_\perp, v_\parallel) - \frac{1}{v_T^2} e^{-\frac{v_\perp^2}{2v_T^2}} \hat{L}^{-1}(v_\perp, v_\parallel)\, \alpha \times \int_0^\infty dv_\perp' v_\perp' \int_{-\infty}^{+\infty} dv_\parallel' \, \beta(v_\parallel') f_{\vec{m}}(v_\perp', v_\parallel') =$$

$$= \hat{L}^{-1}(v_\perp, v_\parallel) Q(v_\perp)$$

Apply integration over $v_\parallel$ and $v_\perp$:

$$\int_0^\infty dv_\perp v_\perp \int_{-\infty}^{+\infty} dv_\parallel \, \beta(v_\parallel) f_{\vec{m}}(v_\perp, v_\parallel) - \frac{1}{v_T^2} \int_0^\infty dv_\perp v_\perp e^{-\frac{v_\perp^2}{2v_T^2}} \int_{-\infty}^{+\infty} dv_\parallel \, \beta(v_\parallel) \hat{L}^{-1}(v_\perp, v_\parallel)\alpha \times$$

$$\times \int_0^\infty dv_\perp' v_\perp' \int_{-\infty}^{+\infty} dv_\parallel' \, \beta(v_\parallel') f_{\vec{m}}(v_\perp', v_\parallel') = \int_0^\infty dv_\perp v_\perp \int_{-\infty}^{+\infty} dv_\parallel \, \beta(v_\parallel) \hat{L}^{-1}(v_\perp, v_\parallel) Q(v_\perp)$$

$$\int_0^\infty dv_\perp' v_\perp' \int_{-\infty}^{+\infty} dv_\parallel' \, \beta(v_\parallel') f_{\vec m}(v_\perp', v_\parallel') = \left(1 - \frac{1}{v_T^2} \int_0^\infty dv_\perp' v_\perp' e^{-\frac{v_\perp'^2}{2v_T^2}} \right.$$ ②

$$\left. \int_{-\infty}^{+\infty} dv_\parallel' \beta(v_\parallel') \hat{L}^{-1}(v_\perp', v_\parallel') \alpha \right)^{-1} \int_0^\infty dv_\perp'' v_\perp'' \int_{-\infty}^{+\infty} dv_\parallel'' \beta(v_\parallel'') \hat{L}^{-1}(v_\perp'', v_\parallel'') Q(v_\perp'')$$

Result: (similar to eq. (62))

$$f_{\vec m}(v_\perp, v_\parallel) = \hat{L}^{-1}(v_\perp, v_\parallel) Q(v_\perp) + \frac{1}{v_T^2} e^{-\frac{v_\perp^2}{2v_T^2}} \hat{L}^{-1}(v_\perp, v_\parallel) \alpha \times$$

$$\times \left[1 - \frac{1}{v_T^2} \int_0^\infty dv_\perp' v_\perp' e^{-\frac{v_\perp'^2}{2v_T^2}} \int_{-\infty}^{+\infty} dv_\parallel' \beta(v_\parallel') \hat{L}^{-1}(v_\perp', v_\parallel') \alpha \right]^{-1} \times$$

$$\times \int_0^\infty dv_\perp'' v_\perp'' \int_{-\infty}^{+\infty} dv_\parallel'' \beta(v_\parallel'') \hat{L}^{-1}(v_\perp'', v_\parallel'') Q(v_\perp'') .$$

Now we transform to $(\lambda, \bar{u})$ variables: $v_\parallel = u_\parallel - V_\parallel$ – shifted velocity

$$\alpha(v_\parallel) = \frac{\nu}{\sqrt{2\pi}\, v_T} e^{-\frac{v_\parallel^2}{2v_T^2}} \left(\frac{v_\parallel^2}{v_T^2} - 1 \right)$$

$$v_\parallel = v_T (1+2\alpha)^{-\frac{1}{2}} \bar{u}$$

$$\alpha(\bar{u}) = \frac{\nu}{\sqrt{2\pi}\, v_T} e^{-\frac{1}{2}\frac{\bar{u}^2}{1+2\alpha}} \left(\frac{\bar{u}^2}{1+2\alpha} - 1 \right)$$

$$\beta(v_\parallel) = \frac{v_\parallel^2}{v_T^2} - 1 = \frac{\bar{u}^2}{1+2\alpha} - 1 \quad , \quad \frac{v_\perp}{v_T} = \lambda$$

$$f_{\vec m}(v_\perp, v_\parallel) = f_{\vec m}(\lambda, \bar{u}) = \int_{-\infty}^{+\infty} d\bar{u}' \, \hat{G}(\lambda, \bar{u}, \bar{u}') Q_{\vec m}(\lambda, \bar{u}') +$$

$$+ \frac{1}{v_T^2} e^{-\frac{1}{2}\lambda^2} \int_{-\infty}^{+\infty} d\bar{u}' \, \hat{G}(\lambda, \bar{u}, \bar{u}') \frac{\nu}{\sqrt{2\pi}\, v_T} e^{-\frac{1}{2}\frac{\bar{u}'^2}{1+2\alpha}} \left(\frac{\bar{u}'^2}{1+2\alpha} - 1 \right) \times C_{\vec m} \times$$

$$\int_0^\infty d\lambda'' v_T \lambda'' v_T \int_{-\infty}^{+\infty} d\bar{u}'' v_T (1+2\alpha)^{-\frac{1}{2}} \left(\frac{\bar{u}''^2}{1+2\alpha} - 1 \right) \int_{-\infty}^{+\infty} d\bar{u}''' \, \hat{G}(\lambda'', \bar{u}'', \bar{u}''') Q_{\vec m}(\lambda'', \bar{u}''')$$

$$C_{\vec{m}} = \Big[1 - \frac{1}{v_T^2}\int_0^{\infty} d\lambda' v_T \lambda' v_T e^{-\frac{1}{2}\lambda'^2} \int_{-\infty}^{+\infty} d\bar{u}' v_T (1+2\alpha)^{-\frac{1}{2}} \left(\frac{\bar{u}'^2}{1+2\alpha} - 1\right)\cdot$$

$$\int_{-\infty}^{+\infty} d\bar{u}'' \hat{G}(\lambda', \bar{u}', \bar{u}'') \frac{\nu}{\sqrt{2\pi}\, v_T} e^{-\frac{1}{2}\frac{\bar{u}''^2}{1+2\alpha}} \left(\frac{\bar{u}''^2}{1+2\alpha} - 1\right)\Big]^{-1} =$$

$$= \Big[1 - \frac{\nu}{\sqrt{2\pi}} \int_0^{\infty} d\lambda' \lambda' e^{-\frac{1}{2}\lambda'^2} \int_{-\infty}^{+\infty} d\bar{u}' (1+2\alpha)^{-\frac{1}{2}} \int_{-\infty}^{+\infty} d\bar{u}'' \hat{G}(\lambda', \bar{u}', \bar{u}'')\times$$

$$\cdot e^{-\frac{1}{2}\frac{\bar{u}''^2}{1+2\alpha}} \left(\frac{\bar{u}'^2}{1+2\alpha} - 1\right)\left(\frac{\bar{u}''^2}{1+2\alpha} - 1\right)\Big]^{-1} =$$

$$= \Big[1 - \frac{\nu}{\sqrt{2\pi}} \int_0^{\infty} d\lambda' \lambda' e^{-\frac{1}{2}\lambda'^2} (1+2\alpha)^{-\frac{1}{2}} \int_{-\infty}^{+\infty} d\bar{u}' \int_{-\infty}^{+\infty} d\bar{u}'' \hat{G}(\lambda', \bar{u}', \bar{u}'')\cdot$$

$$e^{-\frac{1}{2}\frac{\bar{u}''^2}{1+2\alpha}} \left(\frac{\bar{u}'^2 \bar{u}''^2}{(1+2\alpha)^2} - \frac{\bar{u}''^2}{1+2\alpha} - \frac{\bar{u}'^2}{1+2\alpha} + 1\right)\Big]^{-1}.$$

$$f_{\vec{m}}(\lambda, \bar{u}) = \int_{-\infty}^{+\infty} d\bar{u}' \hat{G}(\lambda, \bar{u}, \bar{u}') Q_{\vec{m}}(\lambda, \bar{u}') + \frac{\nu}{\sqrt{2\pi}} e^{-\frac{1}{2}\lambda^2} \times C_{\vec{m}} \times$$

$$\int_{-\infty}^{+\infty} d\bar{u}''' \hat{G}(\lambda, \bar{u}, \bar{u}''') e^{-\frac{1}{2}\frac{\bar{u}'''^2}{1+2\alpha}} \left(\frac{\bar{u}'''^2}{1+2\alpha} - 1\right) \times \int_0^{\infty} d\lambda'' \lambda'' (1+2\alpha)^{-\frac{1}{2}}\cdot$$

$$\cdot \int_{-\infty}^{+\infty} d\bar{u}'' \left(\frac{\bar{u}''^2}{1+2\alpha} - 1\right) \int_{-\infty}^{+\infty} d\bar{u}' \hat{G}(\lambda'', \bar{u}'', \bar{u}') Q_{\vec{m}}(\lambda'', \bar{u}')$$

We write solution as

$$f_{\vec{m}}(\lambda, \bar{u}) = \int_0^{\infty} d\lambda' \int_{-\infty}^{+\infty} d\bar{u}' \hat{G}_{\vec{m}p}(\lambda, \bar{u}, \lambda', \bar{u}') Q_{\vec{m}}(\lambda', \bar{u}'),$$

$$\hat{G}_{\vec{m}p}(\lambda,\bar{u},\lambda',\bar{u}') = \hat{G}(\lambda,\bar{u},\bar{u}')\,\delta(\lambda-\lambda') +$$

$$+\, C_{\vec{m}} \overset{\text{included}}{\frac{\nu}{\sqrt{2\pi}}}\, e^{-\frac{1}{2}\lambda^2} \int_{-\infty}^{+\infty} d\bar{u}'''\, \hat{G}(\lambda,\bar{u},\bar{u}''')\, e^{-\frac{1}{2}\frac{\bar{u}'''^2}{1+2\alpha}} \left(\frac{\bar{u}'''^2}{1+2\alpha}-1\right) \times$$

$$\times\, \lambda' (1+2\alpha)^{-\frac{1}{2}} \int_{-\infty}^{+\infty} d\bar{u}''\, \hat{G}(\lambda',\bar{u}'',\bar{u}') \left(\frac{\bar{u}''^2}{1+2\alpha}-1\right) \cancel{Q_{\vec{m}}(\lambda',\bar{u}')}.$$

Now we introduce special functions:

$$I^{mn}(\lambda) = \frac{\nu}{\sqrt{2\pi}} (1+2\alpha)^{-\frac{m+n+1}{2}} \int_{-\infty}^{+\infty} d\bar{u} \int_{-\infty}^{+\infty} d\bar{u}'\, \bar{u}^m \bar{u}'^n e^{-\frac{1}{2}\frac{\bar{u}'^2}{1+2\alpha}}\, \hat{G}(\lambda,\bar{u},\bar{u}')$$

$$I_\ell^{mn} = \left[\int_0^\infty d\lambda\, \lambda^\ell e^{-\frac{1}{2}\lambda^2}\right]^{-1} \int_0^\infty d\lambda\, \lambda^\ell e^{-\frac{1}{2}\lambda^2} I^{mn}(\lambda) = \left[2^{\frac{\ell-1}{2}}\Gamma\left(\frac{\ell+1}{2}\right)\right]^{-1} \times$$

$$\times\, \frac{\nu}{\sqrt{2\pi}} (1+2\alpha)^{-\frac{m+n+1}{2}} \int_0^\infty d\lambda \int_{-\infty}^{+\infty} d\bar{u} \int_{-\infty}^{+\infty} d\bar{u}'\, \lambda^\ell e^{-\frac{1}{2}\lambda^2} \bar{u}^m \bar{u}'^n e^{-\frac{1}{2}\frac{\bar{u}'^2}{1+2\alpha}}\, \hat{G}(\lambda,\bar{u},\bar{u}').$$

$$I_{k\ell}^{mn} = \frac{\nu}{\sqrt{2\pi}} (1+2\alpha)^{-\frac{m+n+1}{2}} \left[2^{\frac{k+\ell}{2}}\Gamma\left(\frac{k+\ell+2}{2}\right)\right]^{-1} \times$$

$$\times \int_0^\infty d\lambda \int_{-\infty}^{+\infty} d\bar{u} \int_0^\infty d\lambda' \int_{-\infty}^{+\infty} d\bar{u}'\, \lambda^{k+1} \lambda'^\ell \bar{u}^m \bar{u}'^n e^{-\frac{1}{2}\frac{\bar{u}'^2}{1+2\alpha}} e^{-\frac{1}{2}\lambda'^2}\, G_{\vec{m}p}(\lambda,\bar{u},\lambda',\bar{u}').$$

Let we evaluate $C_{\vec{m}}$ constant (with $\frac{\nu}{\sqrt{2\pi}}$ included):

$$C_{\vec{m}} = \frac{\nu}{\sqrt{2\pi}} \Big[1 - \frac{\nu}{\sqrt{2\pi}} (1+2\alpha)^{-\frac{1}{2}} \Big\{ (1+2\alpha)^{-2} \cdot I_1^{22} \cdot \frac{\sqrt{2\pi}}{\nu} (1+2\alpha)^{\frac{2+2+1}{2}} -$$

$$- (1+2\alpha)^{-1} I_1^{02} \frac{\sqrt{2\pi}}{\nu} \cdot (1+2\alpha)^{\frac{0+2+1}{2}} - (1+2\alpha)^{-1} I_1^{20} \frac{\sqrt{2\pi}}{\nu} (1+2\alpha)^{+\frac{2+0+1}{2}} + I_1^{00} \frac{\sqrt{2\pi}}{\nu} \times$$

$$\times (1+2\alpha)^{\frac{0+0+1}{2}} \Big\} \Big]^{-1} = \frac{\nu}{\sqrt{2\pi}} \left[1 - I_1^{22} + I_1^{02} + I_1^{20} - I_1^{00} \right]^{-1}.$$

$$\cancel{C_{\vec{m}} = \frac{\nu}{\sqrt{2\pi}} \left(1 + I_1^{00} + 2 I_1^{02} - I_1^{22}\right)^{-1}}$$

$$C_{\vec m} = \frac{\nu}{\sqrt{2\pi}} \left[1 - I_1^{00} + 2 I_1^{02} - I_1^{22} \right]^{-1}$$

$$I_{kl}^{mn} = I_{k+l+1}^{mn} + \frac{\nu}{\sqrt{2\pi}} (1+2\alpha)^{-\frac{m+n+1}{2}} \left[2^{\frac{k+l}{2}} \Gamma\left(\frac{k+l+2}{2}\right)\right]^{-1} \times$$

$$\times\, C_{\vec m} \int_0^\infty d\lambda \int_{-\infty}^{+\infty} d\bar u \int_0^\infty d\lambda' \int_{-\infty}^{+\infty} d\bar u' \, \lambda^{k+1} \lambda'^{l} e^{-\frac12 \lambda'^2} \bar u^m \bar u'^n e^{-\frac12 \frac{\bar u'^2}{1+2\alpha}} \cdot$$

$$e^{-\frac12 \lambda^2} \int_{-\infty}^{+\infty} d\bar u''' \, \hat G(\lambda, \bar u, \bar u''')\, e^{-\frac12 \frac{\bar u'''^2}{1+2\alpha}} \left(\frac{\bar u'''^2}{1+2\alpha} - 1\right) \times \lambda' (1+2\alpha)^{-\frac12} \times$$

$$\int_{-\infty}^{+\infty} d\bar u'' \, \hat G(\lambda', \bar u'', \bar u') \left(\frac{\bar u''^2}{1+2\alpha} - 1\right) = I_{k+l+1}^{mn} + \frac{\nu}{\sqrt{2\pi}} (1+2\alpha)^{-\frac{m+n+1}{2}}$$

$$\left[2^{\frac{k+l}{2}} \Gamma\left(\frac{k+l+2}{2}\right)\right]^{-1} C_{\vec m} \int_0^\infty d\lambda \int_{-\infty}^{+\infty} d\bar u \int_{-\infty}^{+\infty} d\bar u''' \, \lambda^{k+1} \bar u^m e^{-\frac12 \lambda^2} \hat G(\lambda, \bar u, \bar u''')$$

$$e^{-\frac12 \frac{\bar u'''^2}{1+2\alpha}} \left(\frac{\bar u'''^2}{1+2\alpha} - 1\right) \times \int_0^\infty d\lambda' \int_{-\infty}^{+\infty} d\bar u' \int_{-\infty}^{+\infty} d\bar u'' \, \lambda'^{l+1} e^{-\frac12 \lambda'^2} \bar u'^n e^{-\frac12 \frac{\bar u'^2}{1+2\alpha}}$$

$$(1+2\alpha)^{-\frac12} \hat G(\lambda', \bar u'', \bar u') \left(\frac{\bar u''^2}{1+2\alpha} - 1\right) = I_{k+l+1}^{mn} + \frac{\nu}{\sqrt{2\pi}} (1+2\alpha)^{-\frac{m+n+1}{2}}$$

$$\left[2^{\frac{k+l}{2}} \Gamma\left(\frac{k+l+2}{2}\right)\right]^{-1} C_{\vec m} \cdot \left[2^{\frac{k}{2}} \Gamma\left(\frac{k+2}{2}\right)\right] \cdot \frac{\sqrt{2\pi}}{\nu} \cancel{(1+2\alpha)^{m+}} \times$$

$$\times \left[(1+2\alpha)^{-1} (1+2\alpha)^{\frac{m+2+1}{2}} I_{k+1}^{m2} - (1+2\alpha)^{\frac{m+0+1}{2}} I_{k+1}^{m0} \right] \times$$

$$\times\, (1+2\alpha)^{-\frac12} \cdot \left[2^{\frac{l}{2}} \Gamma\left(\frac{l+2}{2}\right)\right] \frac{\sqrt{2\pi}}{\nu} \left[(1+2\alpha)^{-1} (1+2\alpha)^{\frac{2+n+1}{2}} I_{l+1}^{2n} - \right.$$

$$\left. - (1+2\alpha)^{\frac{0+n+1}{2}} I_{l+1}^{0n} \right] = I_{k+l+1}^{mn} + (1+2\alpha)^{-\frac{m+n+1}{2}} \left[2^{\frac{k+l}{2}} \Gamma\left(\frac{k+l+2}{2}\right)\right]^{-1}$$

$$\left[1 - I_1^{00} + 2 I_1^{02} - I_1^{22} \right]^{-1} \cdot 2^{\frac{k+l}{2}} \Gamma\left(\frac{k+2}{2}\right) \Gamma\left(\frac{l+2}{2}\right) (1+2\alpha)^{-\frac12} \times$$

$$\times\ (1+2\alpha)^{\frac{m+\frac{1}{2}}{2}}\left[I^{m2}_{k+1}-I^{m0}_{k+1}\right]\times(1+2\alpha)^{\frac{n+\frac{1}{2}}{2}}\left[I^{2n}_{\ell+1}-I^{0n}_{\ell+1}\right]=$$

$$=I^{mn}_{k+\ell+1}+\frac{\Gamma\left(\frac{k}{2}+1\right)\Gamma\left(\frac{\ell}{2}+1\right)}{\Gamma\left(\frac{k+\ell}{2}+1\right)}\frac{\left(I^{m2}_{k+1}-I^{m0}_{k+1}\right)\left(I^{n2}_{\ell+1}-I^{n0}_{\ell+1}\right)}{1-I^{00}_1+2I^{02}_1-I^{22}_1}.$$

Conductivity

$$\tilde{f}_{\vec{m}}(\vec{J},t)=\tilde{f}_{\vec{m}}(\lambda,r_0,\bar{u})=\int_0^{\infty}d\lambda'\int_{-\infty}^{+\infty}d\bar{u}'\ \hat{G}_{\vec{m}p}(\lambda,\bar{u},\lambda',\bar{u}')\,Q_{\vec{m}}(\lambda',\bar{u}')$$

$$Q_{\vec{m}}(\lambda',\bar{u}')=-\frac{e}{\omega}\cdot\delta_{m_\vartheta k_\vartheta}\,\delta_{m_z k_z}\,e^{-i\omega t}\left(\sum_{n=0}^{N}\left[a^j_\alpha(n)\right]_e\frac{\partial^n}{\partial r_g^n}\right)\tilde{E}_j(r_g)\times$$

$$\times\left[\left(\vec{m}\cdot\frac{\partial f_0}{\partial\vec{J}}\right)\Omega^\alpha-(\vec{m}\cdot\vec{\Omega}-\omega)\frac{\partial f_0}{\partial J_\alpha}\right]\Bigg|_{(\lambda',\bar{u}')}$$

$$\tilde{f}_{\vec{m}}(\lambda,r_0,\bar{u})=-\frac{e}{\omega}\,\delta_{m_\vartheta k_\vartheta}\,\delta_{m_z k_z}\,e^{-i\omega t}\int_0^{\infty}d\lambda'\int_{-\infty}^{+\infty}d\bar{u}'\ \hat{G}_{\vec{m}p}(\lambda,\bar{u},\lambda',\bar{u}')\cdot$$

$$\cdot\left(\sum_{n=0}^{N}\left[a^j_\alpha(n)\right]_e\frac{\partial^n}{\partial r_g^n}\right)\tilde{E}_j(r_g)\left[\left(\vec{m}\cdot\frac{\partial f_0}{\partial\vec{J}}\right)\Omega^\alpha-(\vec{m}\cdot\vec{\Omega}-\omega)\frac{\partial f_0}{\partial J_\alpha}\right]_{(\lambda',\bar{u}')}$$

$$\tilde{j}^{\,k}(\vec{x},t)=\frac{e}{r}\int_{-\pi}^{+\pi}d\phi\,d\vartheta_g\,dz_g\int\underbrace{\frac{m_0v_T^2}{\omega_c}\lambda\,d\lambda}_{dJ_\perp}\ y\,\underbrace{dr_0\,v_T(1+2\alpha)^{-\frac{1}{2}}d\bar{u}}_{dr_0\,du_{\parallel}}$$

$$\delta(r-r_0-\rho^r)\,\delta(\vartheta-\vartheta_g-\rho^\vartheta)\,\delta(z-z_g-\rho^z)\,\frac{\partial x_c^k}{\partial\theta^\alpha}\,\Omega^\alpha(\lambda,\bar{u})\times$$

$$\times\sum_{\ell=-\infty}^{+\infty}e^{i\ell\phi}\,e^{ik_\vartheta\vartheta_g+ik_zz_g}\,e^{-i\omega t}\left(-\frac{e}{\omega}\right)\int_0^{\infty}d\lambda'\int_{-\infty}^{+\infty}d\bar{u}'\ \hat{G}_{\vec{m}p}(\lambda,\bar{u},\lambda',\bar{u}')$$

$$\left(\sum_{n=0}^{N}\left[a^j_\alpha(n)\right]_e\frac{\partial^n}{\partial r_g^n}\right)\tilde{E}_j(r_g)\left[\left(\vec{m}\cdot\frac{\partial f_0}{\partial\vec{J}}\right)\Omega^\alpha-(\vec{m}\cdot\vec{\Omega}-\omega)\frac{\partial f_0}{\partial J_\alpha}\right]_{(\lambda',\bar{u}')}=$$

$$= \frac{e}{r} \int_{-\pi}^{+\pi} d\phi \left(\frac{m_0 V_T}{\omega_c}\right)^2 (1+2\alpha)^{-\frac{1}{2}} \sum_e e^{i\ell\phi} \cdot e^{ik_\vartheta(\vartheta-\rho^\vartheta)+ik_z(z-\rho^z)} \cdot e^{-i\omega t} \cdot$$ (7)

$$\int d\lambda \int dr_0\, \lambda\, \mathcal{J}\, V_T \int d\bar{u}\ \delta(r-r_0-\rho^r)\ \frac{\partial x_c^k}{\partial\theta^\alpha}\, \Omega^\alpha(\lambda,\bar{u}) \times \left(-\frac{e}{\omega}\right) *$$

$$\int_0^\infty d\lambda' \int_{-\infty}^{+\infty} d\bar{u}' \ldots = -\frac{e^2}{\omega r}\frac{m_0 V_T^3}{\omega_c}(1+2\alpha)^{-\frac{1}{2}} e^{ik_\vartheta\vartheta+ik_z z-i\omega t} \times$$

$$\times \sum_e \int_{-\pi}^{+\pi} d\phi\, e^{i\ell\phi}\, e^{-ik_\vartheta\rho^\vartheta-ik_z\rho^z} \int d\lambda \int dr_0 \int d\bar{u}\ \mathcal{J}\lambda\ \delta(r-r_0-\rho^r)\cdot$$

$$\frac{\partial x_c^k}{\partial\theta^\alpha}\Omega^\alpha(\lambda,\bar{u}) \int_0^\infty d\lambda' \int_{-\infty}^{+\infty} d\bar{u}' \ldots = -\frac{e^2}{\omega r}\frac{m_0 V_T^3}{\omega_c}(1+2\alpha)^{-\frac{1}{2}} \times e^{\cdots} \times$$

$$\sum_e \cancel{\int_{-\pi}^{+\pi} d\phi} \int_0^\infty d\lambda \int_{-\infty}^{+\infty} d\bar{u} \int_0^\infty dr_0 \int_{-\pi}^{+\pi} d\phi\ e^{i\ell\phi}\, e^{-ik_\vartheta\rho^\vartheta-ik_z\rho^z} \times$$

$$\times\left[\frac{\partial X_g^k}{\partial\theta^\alpha} + \frac{\partial\rho^k}{\partial\theta^\alpha} - \delta_{hN}\delta_r^k\frac{\partial\rho^r}{\partial\theta^\alpha}\right] \cdot \sum_{h'=0}^{N}(-)^{h'}\frac{(\rho^r)^{h'}}{h'!}\frac{\partial^{h'}}{\partial r^{h'}}\delta(r-r_0) \times$$

$$\times \mathcal{J}\lambda\ \Omega^\alpha(\lambda,\bar{u}) \int_0^\infty d\lambda' \int_{-\infty}^{+\infty} d\bar{u}' \ldots = -\frac{e^2}{\omega r}\sum_{h'=0}^{N}(-)^{h'}\frac{\partial^{h'}}{\partial r^{h'}}\, e^{\cdots}$$

$$\frac{m_0 V_T^3}{\omega_c}(1+2\alpha)^{-\frac{1}{2}} \sum_e \int_0^\infty d\lambda \int_{-\infty}^{+\infty} d\bar{u}\ \underbrace{\int_{-\pi}^{+\pi} d\phi \left(e^{-i\ell\phi+ik_\vartheta\rho^\vartheta+ik_z\rho^z}[\ldots]\frac{(\rho^r)^{h'}}{h'!}\right)^*}_{2\pi[a_\alpha^k(h')]_e^*}$$

$$\cdot\ \mathcal{J}\cdot\lambda\ \Omega^\alpha(\lambda,\bar{u}) \int_0^\infty d\lambda' \int_{-\infty}^{+\infty} d\bar{u}' \ldots *$$

$$\cancel{j^k(\vec{x})}\ \tilde{j}^k(\vec{x},t) = -\frac{e^2 2\pi}{\omega r}\sum_{h'=0}^{N}(-)^{h'}\frac{\partial^{h'}}{\partial r^{h'}}\ \frac{m_0 V_T^3}{\omega_c}(1+2\alpha)^{-\frac{1}{2}}\sum_e$$

$$\int_0^\infty d\lambda\,\lambda \int_{-\infty}^{+\infty} d\bar{u}\ [a_\alpha^k(h')]_e^* \cdot \mathcal{J}\cdot\Omega^\alpha(\lambda,\bar{u}) \int_0^\infty d\lambda' \int_{-\infty}^{+\infty} d\bar{u}'\ \hat{G}_{\vec{m}p}(\lambda,\bar{u},\lambda',\bar{u}')$$

$$\sum_{h=0}^{N}[a_\beta^j(h)]_e \frac{\partial^h \tilde{E}_j(\vec{x},t)}{\partial r^h}\left[\left(\vec{m}\cdot\frac{\partial f_0}{\partial\vec{J}}\right)\Omega^\beta - (\vec{m}\cdot\vec{\Omega}-\omega)\frac{\partial f_0}{\partial J_\beta}\right]_{(\lambda',\bar{u}')} =$$

$h \leftrightarrow h'$

$$\tilde{J}^k(\vec{x},t) = \frac{2\pi i e^2}{\omega r}\sum_{n=0}^{N}\sum_{n'=0}^{N} (-)^n \frac{\partial^n}{\partial r^n}\,\sigma^{kj}_{nn'}\,\frac{\partial^{n'}}{\partial r^{n'}}\,\tilde{E}_j(\vec{x},t)$$

$$\sigma^{kj}_{nn'}(r,\vec{k}) = i\,\frac{m_0 v_T^3}{\omega_c}(1+2\alpha)^{-\frac{1}{2}}\sum_{\ell}\int_0^{\infty} d\lambda\,\lambda\int_{-\infty}^{+\infty} d\bar{u}\int_0^{\infty} d\lambda'\int_{-\infty}^{+\infty} d\bar{u}'\;\underline{y}\,\cdot$$

$$\underbrace{[a^k_\alpha(n)]^*_\ell\,\Omega^\alpha(\lambda,\bar{u})}_{(\lambda,\bar{u})}\;\underbrace{[a^j_\beta(n')]_\ell\left[\left(\vec{m}\cdot\frac{\partial f_0}{\partial \vec{J}}\right)\Omega^\beta - (\vec{m}\cdot\vec{\Omega}-\omega)\frac{\partial f_0}{\partial J_\beta}\right]}_{(\lambda',\bar{u}')}\times$$

$$\times\;\underbrace{\hat{G}_{J\vec{m}p}(\lambda,\bar{u},\lambda',\bar{u}')}.$$

$$y = m_0^2\, r\,\omega_c$$

$$[a^k_\alpha(n)]^*_\ell = \frac{(-i)^n}{n!}\,A^k_\alpha(n)\,(-i)^{s_1(k,\alpha)}\,(-i)^{s_2(k,\alpha)}\,\frac{\partial^{s_1(k,\alpha)}}{\partial q_1^{s_1(k,\alpha)}}\,\frac{\partial^{s_2(k,\alpha)+n}}{\partial q_2^{s_2(k,\alpha)+n}}\,J_\ell(\tilde{\varkappa}\rho_L\lambda)\,e^{-i\ell\tilde{\alpha}}$$

$$[a^j_\beta(n')]_\ell = \frac{i^{n'}}{n'!}\,A^j_\beta(n')\,i^{s_1(j,\beta)+s_2(j,\beta)}\,\frac{\partial^{s_1(j,\beta)}}{\partial p_1^{s_1(j,\beta)}}\,\frac{\partial^{s_2(j,\beta)+n'}}{\partial p_2^{s_2(j,\beta)+n'}}\,J_\ell(\varkappa\rho_L\lambda')\,e^{i\ell\alpha}$$

$$\Omega^\alpha(\lambda,\bar{u}) = Q^\alpha_{00} + Q^\alpha_{10}\,\bar{u} + Q^\alpha_{20}\,\bar{u}^2 + Q^\alpha_{02}\,\lambda^2$$

$$\left[\left(\vec{m}\cdot\frac{\partial f_0}{\partial \vec{J}}\right)\Omega^\beta - (\vec{m}\cdot\vec{\Omega}-\omega)\frac{\partial f_0}{\partial J_\beta}\right] = \sum_{\mu,\nu} F^\beta_{\mu\nu}(\ell)\,\bar{u}'^{\mu}\lambda'^{2\nu}\times y^{-1}\left(-\frac{f_0}{1}\right)_{(\lambda',\bar{u}')}$$

$$\sigma^{kj}_{nn'} = i\,\frac{m_0 v_T^3}{\omega_c}(1+2\alpha)^{-\frac{1}{2}}\sum_{\ell}\left(\text{factors of } [a]_\ell\right)\int_0^{\infty} d\lambda\int_0^{\infty} d\lambda'\int_{-\infty}^{+\infty} d\bar{u}\int_{-\infty}^{+\infty} d\bar{u}'$$

$$\lambda\, J_\ell(\tilde{\varkappa}\rho_L\lambda)\, J_\ell(\varkappa\rho_L\lambda')\, e^{i\ell(\alpha-\tilde{\alpha})}\left[Q^\alpha_{00} + Q^\alpha_{10}\,\bar{u} + Q^\alpha_{20}\,\bar{u}^2 + Q^\alpha_{02}\,\lambda^2\right]\times$$

$$\times\sum_{\mu,\nu} F^\beta_{\mu\nu}(\ell)\,\bar{u}'^{\mu}\lambda'^{2\nu}\,(-)\,\frac{1}{(2\pi)^{3/2}}\,\frac{n_0}{m_0^3 v_T^3}\,e^{-\frac{1}{2}\lambda'^2}\,e^{-\frac{1}{2}\frac{\bar{u}'^2}{1+2\alpha}}\,\hat{G}_{\vec{m}p}(\lambda,\bar{u},\lambda',\bar{u}')$$

$$\mathcal{J}_\ell(\tilde{\varkappa}\rho_L\lambda) = \sum_{\tilde{b}=0}^{Nb} \frac{(-)^{\tilde{b}}}{\tilde{b}!(\ell+\tilde{b})!}\left(\frac{1}{2}\right)^{\ell+2\tilde{b}}(\tilde{\varkappa}\rho_L)^{\ell+2\tilde{b}}\lambda^{\ell+2\tilde{b}}$$

$$\mathcal{J}_\ell(\varkappa\rho_L\lambda') = \sum_{b=0}^{Nb}\frac{(-)^b}{b!(\ell+b)!}\left(\frac{1}{2}\right)^{\ell+2b}(\varkappa\rho_L)^{\ell+2b}\lambda'^{\,\ell+2b}$$

For $\ell \geq 0$! For $\ell<0$ we use $\mathcal{J}_{-\ell} = (-)^\ell \mathcal{J}_\ell$.

$$\sigma^{kj}_{nn'} = -i\frac{1}{(2\pi)^{3/2}}\frac{m_0 \cancel{V_T^3}}{\omega_c}\frac{n_0}{m_0^3\cancel{V_T^3}}(1+2\alpha)^{-\frac{1}{2}}\sum_\ell{}'(\text{factors of } [a]_c)$$

$$\int_0^\infty d\lambda\int_0^\infty d\lambda'\int_{-\infty}^{+\infty}d\bar{u}\int_{-\infty}^{+\infty}d\bar{u}'\,\lambda\sum_{b,\tilde{b}=0}^{Nb}\frac{(-)^{b+\tilde{b}}}{b!\tilde{b}!(|\ell|+b)!(|\ell|+\tilde{b})!}\left(\frac{1}{2}\right)^{2|\ell|+2b+2\tilde{b}} \times$$

$$\times e^{i\ell\alpha}(\varkappa\rho_L)^{|\ell|+2b}\times e^{-i\ell\tilde{\alpha}}(\tilde{\varkappa}\rho_L)^{|\ell|+2\tilde{b}}\times\lambda'^{\,|\ell|+2b}\times\lambda^{|\ell|+2\tilde{b}}$$

$$\left[Q^\alpha_{00}+Q^\alpha_{10}\bar{u}+Q^\alpha_{20}\bar{u}^2+Q^\alpha_{02}\lambda^2\right]\sum_{\mu,\nu}F^\beta_{\mu\nu}\bar{u}'^{\mu}\lambda'^{2\nu}e^{-\frac{1}{2}\lambda'^2}e^{-\frac{1}{2}\frac{\bar{u}'^2}{1+2\alpha}}\cdot\hat{G} =$$

$$= -\frac{i}{(2\pi)^{3/2}}\frac{n_0}{m_0^2\omega_c}(1+2\alpha)^{-\frac{1}{2}}\sum_\ell{}'\sum_{b,\tilde{b}=0}^{Nb}\frac{(-)^{b+\tilde{b}}2^{-2|\ell|-2b-2\tilde{b}}}{b!\tilde{b}!(|\ell|+b)!(|\ell|+\tilde{b})!}\times$$

$$\times\frac{i^{n'}}{n'!}A^j_\beta(n')\,i^{s_1(j,\beta)+s_2(j,\beta)}\frac{\partial^{s_1(j,\beta)}}{\partial p_1^{s_1(j,\beta)}}\frac{\partial^{s_2(j,\beta)+n'}}{\partial p_2^{s_2(j,\beta)+n'}}e^{i\ell\alpha}(\varkappa\rho_L)^{|\ell|+2b}\times$$

$$\times\frac{(-i)^n}{n!}A^k_\alpha(n)(-i)^{s_1(k,\alpha)+s_2(k,\alpha)}\frac{\partial^{s_1(k,\alpha)}}{\partial q_1^{s_1(k,\alpha)}}\frac{\partial^{s_2(k,\alpha)+n}}{\partial q_2^{s_2(k,\alpha)+n}}e^{-i\ell\tilde{\alpha}}(\tilde{\varkappa}\rho_L)^{|\ell|+2\tilde{b}}\times$$

$$\int_0^\infty d\lambda\int_{-\infty}^{+\infty}d\bar{u}\int_0^\infty d\lambda'\int_{-\infty}^{+\infty}d\bar{u}'\,\lambda^{|\ell|+2\tilde{b}+1}\lambda'^{\,|\ell|+2b}e^{-\frac{1}{2}\lambda'^2}e^{-\frac{1}{2}\frac{\bar{u}'^2}{1+2\alpha}}\hat{G}_{\bar{m}p}(\lambda,\bar{u},\lambda',\bar{u}')$$

$$\sum_{\mu,\nu}F^\beta_{\mu\nu}\left[Q^\alpha_{00}\bar{u}'^{\mu}\lambda'^{2\nu}+Q^\alpha_{10}\bar{u}\bar{u}'^{\mu}\lambda'^{2\nu}+Q^\alpha_{20}\bar{u}^2\bar{u}'^{\mu}\lambda'^{2\nu}+Q^\alpha_{02}\lambda^2\bar{u}'^{\mu}\lambda'^{2\nu}\right]$$

We introduce the following functions:

$$D_{eb}^{mn}(\kappa_\perp \rho_L) = \rho_L^{-(m+n)} \, i^{m+n} \frac{\partial^m}{\partial \varkappa_1^m} \frac{\partial^n}{\partial \varkappa_2^n} e^{ie\xi} (\varkappa \rho_L)^{|e|+2b} \Big|_{\substack{\varkappa_1 = \kappa_\perp \\ \varkappa_2 = 0}}$$

where $\varkappa = \sqrt{\varkappa_1^2 + \varkappa_2^2}$, $\xi = \operatorname{atan}(\varkappa_2/\varkappa_1)$

$$\sigma_{nn'}^{kj} = - \frac{i}{(2\pi)^{3/2}} \frac{n_0}{m_0^2 \omega_c} (1+2\alpha)^{-\frac{1}{2}} \sum_e{}' \sum_{b,\tilde{b}=0}^{N_b} \frac{(-)^{b+\tilde{b}} \, 2^{-2|e|-2b-2\tilde{b}}}{b!\,\tilde{b}!\,(|e|+b)!\,(|e|+\tilde{b})!} \times$$

$$\frac{A_\beta^j(n')}{n'!} \frac{A_\alpha^k(n)}{n!} \rho_L^{n'+s_1(j,\beta)+s_2(j,\beta)+n+s_1(k,\alpha)+s_2(k,\alpha)} \times$$

$$\times D_{eb}^{s_1(j,\beta)\; s_2(j,\beta)+n'} \times \left[D_{e\tilde{b}}^{s_1(k,\alpha)\; s_2(k,\alpha)+n} \right]^* \times \int_0^\infty d\lambda \int_{-\infty}^{+\infty} d\bar{u} \int_0^\infty d\lambda' \int_{-\infty}^{+\infty} d\bar{u}'$$

$$e^{-\frac{1}{2}\lambda'^2} e^{-\frac{1}{2}\frac{\bar{u}'^2}{1+2\alpha}} \hat{G}_{\hat{m}p}(\lambda, \bar{u}, \lambda', \bar{u}') \sum_{\mu,\nu}{}' F_{\mu\nu}^\beta \Big[Q_{00}^\alpha \underbrace{\bar{u}^0 \bar{u}'^\mu \lambda^{|e|+2\tilde{b}+1}}_{m=0,\, n=\mu,\, k=|e|+2\tilde{b},\, \ell=|e|+2b+2\nu}$$

$$\lambda'^{|e|+2b+2\nu} + Q_{10}^\alpha \underbrace{\bar{u}^1 \bar{u}'^\mu \lambda^{|e|+2\tilde{b}+1} \lambda'^{|e|+2b+2\nu}}_{m=1,\, n=\mu,\, k=|e|+2\tilde{b},\, \ell=|e|+2b+2\nu} + Q_{20}^\alpha \underbrace{\bar{u}^2 \bar{u}'^\mu \lambda^{|e|+2\tilde{b}+1}}_{m=2,\, n=\mu,\, k=|e|+2\tilde{b},\, \ell=|e|+2b+2\nu}$$

$$\lambda'^{|e|+2b+2\nu} + Q_{02}^\alpha \underbrace{\bar{u}^0 \bar{u}'^\mu \lambda^{|e|+2\tilde{b}+1+2} \lambda'^{|e|+2b+2\nu}}_{m=0,\, n=\mu,\, k=|e|+2\tilde{b}+2,\, \ell=|e|+2b+2\nu} \Big] =$$

$$= - \frac{i}{(2\pi)^{3/2}} \frac{n_0}{m_0^2 \omega_c} \frac{A_\beta^j(n')}{n'!} \frac{A_\alpha^k(n)}{n!} \rho_L^{n'+s_1(j,\beta)+s_2(j,\beta)+n+s_1(k,\alpha)+s_2(k,\alpha)} \times$$

$$\sum_e{}' \sum_{b,\tilde{b}=0}^{N_b} \frac{(-)^{b+\tilde{b}} \, 2^{-2|e|-2b-2\tilde{b}}}{b!\,\tilde{b}!\,(|e|+b)!\,(|e|+\tilde{b})!} D_{eb}^{s_1(j,\beta)\; s_2(j,\beta)+n'} \left[D_{e\tilde{b}}^{s_1(k,\alpha)\; s_2(k,\alpha)+n} \right]^*$$

$$\sum_{\mu,\nu}{}' F_{\mu\nu}^\beta \, (1+2\alpha)^{-\frac{1}{2}} \int_0^\infty d\lambda \int_{-\infty}^{+\infty} d\bar{u} \int_0^\infty d\lambda' \int_{-\infty}^{+\infty} d\bar{u}' \; \hat{G}_{\hat{m}p}(\lambda, \bar{u}, \lambda', \bar{u}')$$

$$e^{-\frac{1}{2}\lambda'^2} e^{-\frac{1}{2}\frac{\bar{u}'^2}{1+2\alpha}} \left[Q_{00}^\alpha \cdots \right] =$$

$$= -\frac{i}{(2\pi)^{3/2}} \cdots (1+2\lambda)^{-\frac{1}{2}} \frac{\sqrt{2\pi}}{\nu} \Big[Q^{\alpha}_{00} (1+2\lambda)^{\frac{0+\mu+1}{2}} 2^{\frac{1}{2}(|e|+2\tilde{b}+|e|+2b+2\nu)}$$ (11)

$$\Gamma\left(\tfrac{1}{2}(|e|+2\tilde{b}+|e|+2b+2\nu+2)\right) I^{0\mu}_{|e|+2\tilde{b}\ \ |e|+2b+2\nu} +$$

$$+ Q^{\alpha}_{10} (1+2\lambda)^{\frac{1+\mu+1}{2}} 2^{\frac{1}{2}(|e|+2\tilde{b}+|e|+2b+2\nu)} \Gamma\left(\tfrac{1}{2}(|e|+2\tilde{b}+|e|+2b+2\nu+2)\right) I^{1\mu}_{|e|+2\tilde{b}\ \ |e|+2b+2\nu}$$

$$+ Q^{\alpha}_{20} (1+2\lambda)^{\frac{2+\mu+1}{2}} 2^{\frac{1}{2}(|e|+2\tilde{b}+|e|+2b+2\nu)} \Gamma\left(\tfrac{1}{2}(|e|+2\tilde{b}+|e|+2b+2\nu+2)\right) I^{2\mu}_{|e|+2\tilde{b}\ \ |e|+2b+2\nu}$$

$$+ Q^{\alpha}_{02} (1+2\lambda)^{\frac{0+\mu+1}{2}} 2^{\frac{1}{2}(|e|+2\tilde{b}+2+|e|+2b+2\nu)} \Gamma\left(\tfrac{1}{2}(|e|+2\tilde{b}+2+|e|+2b+2\nu+2)\right) I^{0\mu}_{|e|+2\tilde{b}+2\ \ |e|+2b+2\nu} \Big]$$

$$= -\frac{i}{(2\pi)^{3/2}} \cdots \ / \ Q_{\mu 0} \text{ and } F^{\beta}_{\mu\nu} \text{ contains } (1+2\lambda)^{-\frac{\mu}{2}} \text{ factors inside - we annigilate it!} /$$

$$= -\frac{i}{(2\pi)^{3/2}} \frac{\sqrt{2\pi}}{\nu} \cdots 2^{|e|+b+\tilde{b}+\nu} \underbrace{\Gamma(|e|+b+\tilde{b}+\nu+1)}_{(|e|+b+\tilde{b}+\nu)!} \times$$

$$\Big[Q^{\alpha}_{00} I^{0\mu}_{|e|+2\tilde{b},\,|e|+2b+2\nu} + Q^{\alpha}_{10} I^{1\mu}_{|e|+2\tilde{b},\,|e|+2b+2\nu} + Q^{\alpha}_{20} I^{2\mu}_{|e|+2\tilde{b},\,|e|+2b+2\nu} +$$

$$+ Q^{\alpha}_{02} \cdot 2 \cdot (|e|+b+\tilde{b}+\nu+1) I^{0\mu}_{|e|+2\tilde{b}+2,\,|e|+2b+2\nu} \Big]$$

$$\sigma^{kj}_{nn'} = -\frac{i}{(2\pi)^{3/2}} \frac{\sqrt{2\pi}}{\nu} \frac{n_0}{m_0^2 \omega_c} \cdots\cdots \sum_{\mu,\nu} F^{\beta}_{\mu\nu}$$

$$\sum_{e} \sum_{b,\tilde{b}=0}^{N_{b_1}} \frac{(-)^{b+\tilde{b}}\, 2^{-|e|-b-\tilde{b}+\nu}}{b!\,\tilde{b}!\,(|e|+b)!\,(|e|+\tilde{b})!} \cdot (|e|+b+\tilde{b}+\nu)! \cdot D \cdot [D]^{*}$$

$$[\, Q^{\alpha}_{00} I + \cdots \,]$$

Finally we get:

$$\sigma^{kj}_{nn'} = -\sqrt{2\pi}\,\frac{i}{(2\pi)^{3/2}}\,\frac{h_0}{m_0^2\omega_c}\,\frac{1}{\vartheta}\,\frac{A^j_\beta(n')}{n'!}\,\frac{A^k_\alpha(n)}{n!}\,\rho_i^{\,n'+s_1(j,\beta)+s_2(j,\beta)+n+s_1(k,\alpha)+s_2(k,\alpha)}\times$$

$$\sum_e\sum_{\mu,\nu} F^\beta_{\mu\nu}(e)\sum_{b,\tilde{b}=0}^{N_b}(-)^{b+\tilde{b}}\,2^{-|e|-b-\tilde{b}+\nu}\,\frac{(|e|+b+\tilde{b}+\nu)!}{b!\,\tilde{b}!\,(|e|+b)!\,(|e|+\tilde{b})!}\times$$

$$D^{s_1(j,\beta)\;s_2(j,\beta)+n'}_{eb}\left[D^{s_1(k,\alpha)\;s_2(k,\alpha)+n}_{e\tilde{b}}\right]^*\Big\{Q^\alpha_{00}\,I^{0\mu}_{|e|+2\tilde{b},\,|e|+2b+2\nu}+$$

$$+\,Q^\alpha_{10}\,I^{1\mu}_{|e|+2\tilde{b},\,|e|+2b+2\nu}+Q^\alpha_{20}\,I^{2\mu}_{|e|+2\tilde{b},\,|e|+2b+2\nu}+$$

$$+\,Q^\alpha_{02}\cdot 2\cdot(|e|+b+\tilde{b}+\nu+1)\cdot I^{0\mu}_{|e|+2\tilde{b}+2,\,|e|+2b+2\nu}\Big\}.$$

In ordinary case without energy conservation:

$$I^{mn}_{ke}\to I^{mn}_{k+e+1}.$$